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editorial

Mathematics without Tears

Unlike many subjects such as history, mathematics needs a special method of teaching. To inform, to kindle the thinking and a spirit of extrapolation is all part of teaching mathematics. Let us take for example, the concept of infinity. Add anything to infinity or take away anything from infinity, the sum or difference is only infinity. If infinity is subtracted from infinity, or divided by infinity, the answer is undefined, according to our college mathematics.

However, according to Upanishads, the concept of infinity is i.k&Poorna, Poorna means complete. Subtract infinity from infinity , the answer is infinity according our thinkers. "This is infinity , that is infinity. By taking infinity from infinity, only infinity remains."

When teaching calculus, we should slowly teach the concept of delta *x*. By various examples, such as x, x^2, x^3..., x^n are can teach the concept of differentiation.

Simple methods of teaching can make big concepts very clear. Group theory is used for crystallography. But the mathematics part is the main thing. Integration or differentiation of any complicated function can be performed by numerical methods. Theory of groups is best studied with rangolis. Whatever may be method of teaching, one common thing noticed about any successful teacher is his smiling face with a friendly attitude. A sense of humour can easily reduce the tension of learning for the students.

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Anil Ahlawat
Editor

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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today with the suggestion of Shri Mahabir Singh. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

Prof. Dr. Ramanaiah Gundala, Former Dean of Science and Humanities, Anna University, Chennai

PROBLEM Set 148

JEE MAIN

1. In triangle ABC, if $\cos A \cos B \cos C = -\frac{1}{2}$ and $B = C$, then $C =$

(a) $\cos^{-1}(2\sin 18°)$ (b) $\sin^{-1}(2\cos 36°)$

(c) $\frac{1}{2}\cos^{-1}(2\sin 18°)$ (d) $\frac{1}{2}\sin^{-1}(2\cos 36°)$

2. If $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$, $\vec{b}\cdot\vec{c} = \cos\alpha$, $\vec{c}\cdot\vec{a} = \vec{a}\cdot\vec{b} = \cos\beta$, then

(a) $\alpha \le \beta$ (b) $\beta \le \alpha$ (c) $\alpha \le 2\beta$ (d) $\beta \le 2\alpha$

3. Let $3^{2015} = N$. The sum of the last four digits of N is

(a) 21 (b) 22 (c) 23 (d) 24

4. If a, b, c, d, p are real numbers such that $(a + ib)^{(c + id)} = p > 0$, then $(a^2 + b^2)^{(c^2 + d^2)} =$

(a) p^a (b) p^b (c) p^{2d} (d) p^{2c}

JEE ADVANCED

5. The sum $S = 2016^2 + 2015^2 + 2014^2 - 2013^2 - 2012^2 - 2011^2 + 2010^2 + 2009^2 + 2008^2 - 2007^2 - 2006^2 - 2005^2 + \ldots + 6^2 + 5^2 + 4^2 - 3^2 - 2^2 - 1^2$, is divisible by

(a) 2 (b) 3 (c) 5 (d) 7

6. A straight line is a tangent to the parabola $y^2 = 4x$ and normal to the parabola $x^2 = \sqrt{2}y$. The distance of the origin from the line is

(a) 0 (b) $\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

COMPREHENSION

Let $n \in N$. The G.M. and H.M. of the n numbers $n + 1$, $n + 2$, $n + 3$, …, $n + n$ are G_n and H_n respectively.

7. $\lim_{n\to\infty} \frac{G_n}{n} =$

(a) $\frac{1}{e}$ (b) $\frac{2}{e}$ (c) $\frac{3}{e}$ (d) $\frac{4}{e}$

8. $\lim_{n\to\infty} \frac{H_n}{n} =$

(a) $\frac{1}{e}$ (b) $\frac{1}{\ln 2}$ (c) $\frac{2}{e}$ (d) $\frac{1}{\ln 4}$

INTEGER MATCH

9. If $(10^{2015} + 5)^2 = 225\,N$, then the number of even digits in N is

MATCHING LIST

10.

	Column-I		Column-II
P.	If $x - \frac{1}{x} = i$, then $x^{2015} - \frac{1}{x^{2015}} =$	1.	0
Q.	If $x - \frac{1}{x} = -i$, then $x^{2015} - \frac{1}{x^{2015}} =$	2.	1
R.	If $x - \frac{1}{x} = i$, then $x^{2016} - \frac{1}{x^{2016}} =$	3.	i
S.	If $x - \frac{1}{x} = -i$, then $x^{2014} + \frac{1}{x^{2014}} =$	4.	$-i$

	P	Q	R	S
(a)	1	2	4	3
(b)	2	4	3	1
(c)	4	3	1	2
(d)	3	1	2	4

See Solution set of Maths Musing 147 on page no. 84

Prof. Ramanaiah is the author of MTG JEE(Main & Advanced) Mathematics series

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* ALOK KUMAR, B.Tech, IIT Kanpur

SECTION-I

(Single Correct Answer Type)

This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

1. Let $f(x)=\begin{cases}(x+1)^3, & -2<x\le -1\\ x^{2/3}-1, & -1<x\le 1\\ -(x-1)^2, & 1<x<2\end{cases}$

The total number of maxima and minima of $f(x)$ is

(a) 4 (b) 3 (c) 2 (d) 1

2. Let $x>0$, then $\lim\limits_{x\to 0}(\sqrt{\tan x})^{\sqrt{x}}+(\sec x)^{1/x}=$

(a) $1/e$ (b) 1 (c) $1/e^2$ (d) 2

3. The number of integer values of p for which the vectors $\hat{i}+p^2\hat{j}+p^2\hat{k}$, $p^2\hat{i}+\hat{j}+p^4\hat{k}$ and $p^4\hat{i}+p^4\hat{j}+\hat{k}$ are coplanar is

(a) 8 (b) 4 (c) 2 (d) 0

4. The complete set of values of a for which the point (a, a^2), $a\in R$ lies inside the triangle formed by the lines $x-y+2=0$, $x+y=2$ and x-axis is

(a) $(-2, 2)$ (b) $(-1, 1)$ (c) $(0, 2)$ (d) $(-2, 0)$

5. In a test, student either guesses or copies or knows the answer to a multiple choice questions with four choices in which exactly one choice is correct. The probability that he makes a guess is 1/3 ; the probability that he copies the answer is 1/6. The Probability that his answer is correct, given that he copied it, is 1/8. Find the probability that he knew the answer to the question given that he correctly answered it is

(a) $\frac{29}{35}$ (b) $\frac{24}{29}$ (c) $\frac{1}{7}$ (d) $\frac{1}{9}$

6. A hyperbola has centre C and one focus at $P(6, 8)$. If its two directrices are $3x+4y+10=0$ and $3x+4y-10=0$, then $CP=$

(a) 14 (b) 8 (c) 10 (d) 6

7. If $x^6=2x^3-1$ and x is not real, then $\sum\limits_{r=1}^{50}(x^r+x^{2r})^3=$

(a) 100 (b) 256 (c) 76 (d) 94

8. Consider the system of equations $ax+by=0$ and $cx+dy=0$ where $a, b, c, d\in\{1, 2\}$. The probability that the system of equations has a unique solution is

(a) 3/8 (b) 5/16 (c) 9/16 (d) 5/8

9. Area bounded by the curves $y=e^x$, $y=\log_e x$ and the lines $x=0$, $y=0$, $y=1$ is

(a) e^2+2 sq. units (b) $e+1$ sq. units
(c) $e+2$ sq. units (d) $e-1$ sq. units

10. Let A, B, C be any three events in a sample space of a random experiment. Let the events E_1 = exactly one of A, B occurs; E_2 = exactly one of B, C occurs; E_3 = exactly one of C, A occurs; E_4 = all of A, B, C occurs; E_5 = atleast one of A, B, C occurs. $P(E_1)=P(E_2)=P(E_3)=1/3$; $P(E_4)=1/9$, then $P(E_5)=$

(a) 1/9 (b) 7/9 (c) 5/18 (d) 11/18

11. Let $S(x)=\int\frac{dx}{e^x+8e^{-x}+4e^{-3x}}$, $R(x)=\int\frac{dx}{e^{3x}+8e^x+4e^{-x}}$ and $M(x)=S(x)-2R(x)$.

If $M(x)=\frac{1}{2}\tan^{-1}(f(x))+c$; where c is an arbitrary constant, then $f(\log_e 2)=$

(a) 3/2 (b) 1/2 (c) 5/2 (d) 7/2

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91). He trains IIT and Olympiad aspirants.

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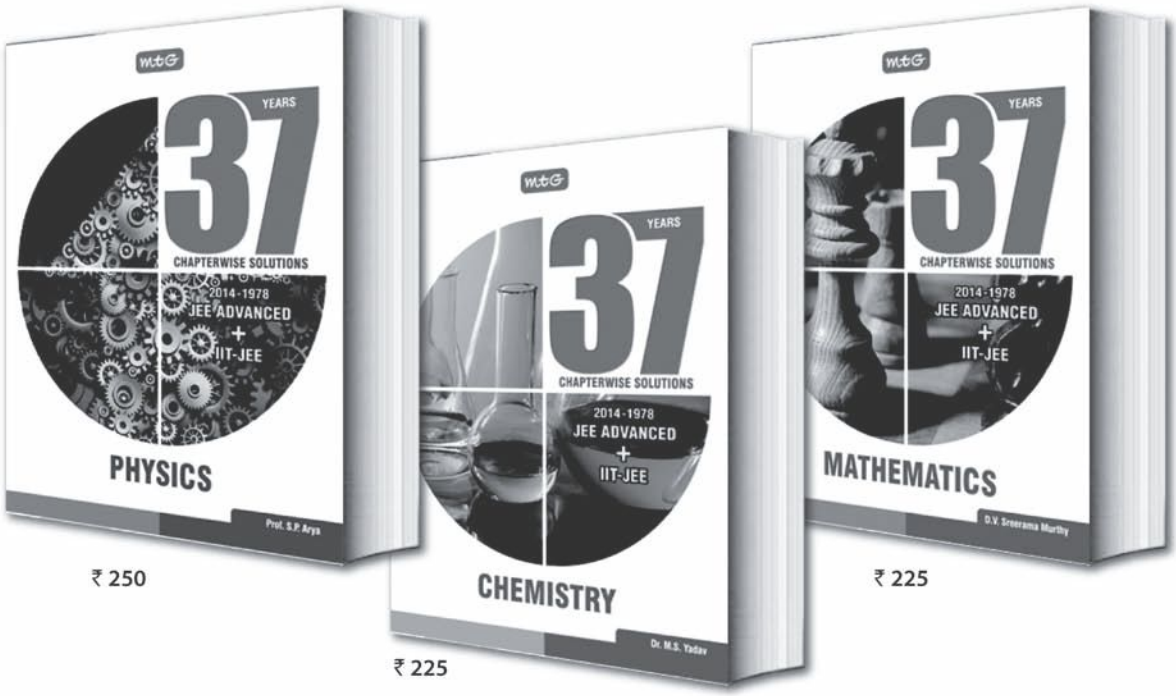

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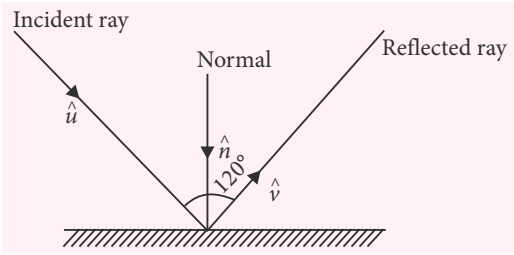
12. A ray of light strikes a plane mirror and gets reflected as shown in the diagram. If $\hat{u}, \hat{v}, \hat{n}$ are unit vectors along the incident ray, reflected ray and normal respectively, then



(a) $\hat{v}-\hat{u}+\hat{n}=\bar{0}$ (b) $\hat{v}+\hat{u}-\sqrt{3}\hat{n}=\bar{0}$

(c) $\hat{v}-\hat{u}+\sqrt{3}\hat{n}=\bar{0}$ (d) $\hat{v}+\hat{u}-3\hat{n}=\bar{0}$

13. Let $L = 0$ be a common normal to the circle $x^2 + y^2 - 2\alpha x - 36 = 0$ and the curve $S : (1 + x)^y + e^{xy} = y$ drawn at a point $x = 0$ on S, then the radius of the circle is

(a) 10 (b) 5 (c) 8 (d) 12

14. If $f(x)=\prod_{i=1}^{i=3}(x-a_i)+\sum_{i=1}^{3}a_i-3x$, where $a_i < a_{i+1}$, then $f(x) = 0$ has

(a) only one real root

(b) three real roots of which two of them are equal

(c) three distinct real roots

(d) three equal roots

15. Let $f:(0,\infty)\to R$ and $F(x)=\int_1^x f(t)dt$.

If $F(x^2) = x^2(1 + x)$, then $f(4)$ equals

(a) 5/4 (b) 7 (c) 4 (d) 2

16. If $f(x)=\int_0^x(1+t^3)^{-1/2}dt$ and $g(x)$ is the inverse of f, then the value of $\frac{g''(x)}{g^2(x)}$ is

(a) 3/2 (b) 2/3 (c) 1/3 (d) 1/2

17. A function $f : R \to R$ satisfies the equation $f(x) f(y) - f(xy) = x + y \ \forall \ x, y \in R$ and $f(1) > 0$, then

(a) $f(x) f^{-1}(x) = x^2 - 4$ (b) $f(x) f^{-1}(x) = x^2 - 6$

(c) $f(x) f^{-1}(x) = x^2 - 1$ (d) none of these

18. The value of 'a' so that the volume of parallelopiped formed by $\hat{i}+a\hat{j}+\hat{k}$, $\hat{j}+a\hat{k}$, and $a\hat{i}+\hat{k}$, becomes minimum is

(a) –3 (b) 3 (c) $1/\sqrt{3}$ (d) $\sqrt{3}$

19. Let $f(x) = x^2 - bx + c$, b an h odd positive integer, $f(x) = 0$ have two prime numbers as roots and $b + c = 35$. Then the global minimum value of $f(x)$ is

(a) $-\frac{183}{4}$ (b) $\frac{173}{16}$

(c) $-\frac{81}{4}$ (d) data not sufficient

20. If A is a skew–symmetric matrix of order 3, then the matrix A^4 is

(a) skew symmetric (b) symmetric

(c) diagonal (d) none of those

21. If $\alpha_1, \alpha_2, ..., \alpha_n$ are roots of the equation $x^n + ax + b = 0$, then $(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)(\alpha_1 - \alpha_4) ... (\alpha_1 - \alpha_n)$ is equal to

(a) n (b) $n\alpha_1^{n-1}$

(c) $n\alpha_1 + b$ (d) $n\alpha_1^{n-1} + a$

22. $x^2 + y^2 + 6x + 8y = 0$ and $x^2 + y^2 - 4x - 6y - 12 = 0$ are the equations of the two circles. Equation of one of their common tangent is

(a) $7x - 5y - 1 - 5\sqrt{74} = 0$

(b) $7x - 5y - 1 + 5\sqrt{74} = 0$

(c) $7x - 5y + 1 - 5\sqrt{74} = 0$

(d) $5x - 7y + 1 - 5\sqrt{74} = 0$

23. Maximum value of $\log_5(3x + 4y)$, if $x^2 + y^2 = 25$ is

(a) 2 (b) 3 (c) 4 (d) 5

24. If A and B are symmetric matrices of same order and $X = AB + BA$ and $Y = AB - BA$, then $(XY)^T$ is equal to

(a) XY (b) YX

(c) $-YX$ (d) none of these

25. The solution of the differential equation

$2x^3ydy + (1 - y^2)(x^2y^2 + y^2 - 1)dx = 0$

[where c is a constant]

(a) $x^2y^2 = (cx + 1)(1 - y^2)$

(b) $x^2y^2 = (cx + 1)(1 + y^2)$

(c) $x^2y^2 = (cx - 1)(1 - y^2)$

(d) none of these

26. The slope of the line which belongs to family of these $(1 + \lambda)x + (\lambda - 1)y + 2(1 - \lambda) = 0$ and makes shortest intercept on $x^2 = 4y - 4$, is

(a) 1/2 (b) 1 (c) 0 (d) 2

27. Sum of integral values of n such that $\sin x(2\sin x + \cos x) = n$, has at least one real solution is

(a) 3 (b) 1 (c) 2 (d) 0

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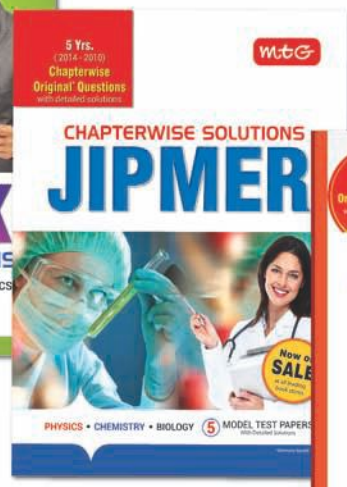

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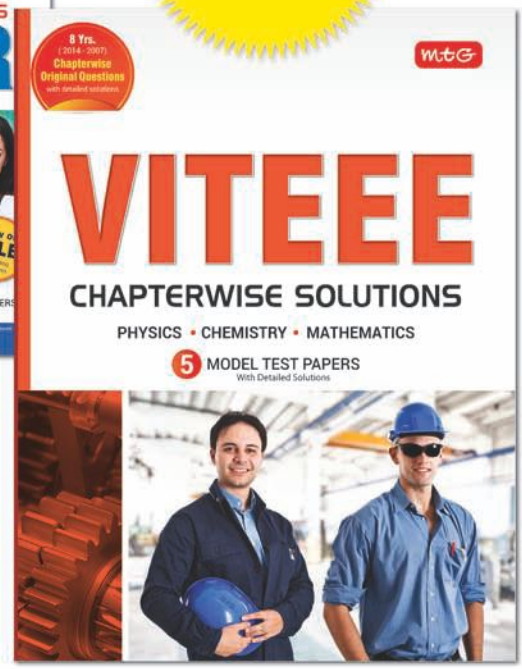

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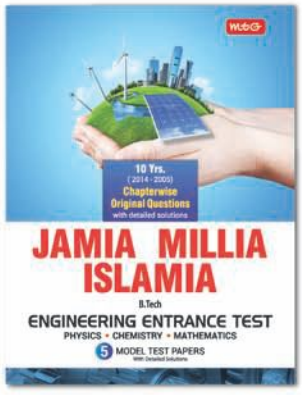

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28. The equation $2x = (2n + 1)\pi(1 - \cos x)$, (where n is a positive integer)
(a) has infinitely many real roots
(b) has exactly one real root
(c) has exactly $2n + 2$ real roots
(d) has exactly $2n + 3$ real roots

29. If the tangents at two points (1, 2) and (3, 6) as a parabola intersect at the point (– 1, 1), then the slope of the directrix of the parabola is
(a) $\sqrt{2}$ (b) -2
(c) -1 (d) none of these

30. Let $f(x)=\dfrac{\tan x}{x}$, then $\log_e\left(\lim\limits_{x\to 0}([f(x)]+x^2)^{\frac{1}{\{f(x)\}}}\right)$ is equal, (where $[\cdot]$ denotes greatest integer function and $\{\cdot\}$ fractional part)
(a) 1 (b) 2 (c) 3 (d) 4

31. If $a_n=\sum\limits_{k=0}^{n}\dfrac{(\log_e 10)^n}{k!(n-k)!}$ for $n \geq 0$ then
$a_0 + a_1 + a_2 + a_3 + \ldots$ upto ∞ is equal
(a) 10 (b) 10^2 (c) 10^3 (d) 10^4

SECTION-II

(Multiple Correct Answer Type)

This section contains multiple correct answer(s) type questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONE OR MORE is/ are correct.

32. In a ΔABC sides b, c, $\angle C$ are given, which of the following cannot determine a unique ΔABC?
(a) $c > b\sin C$, $\angle C < \pi/2$, $c > b$
(b) $c > b\sin C$, $\angle C < \pi/2$, $c < b$
(c) $c > b\sin C$, $\angle C > \pi/2$, $c < b$
(d) $c > b\sin C$, $\angle C < \pi/2$, $b = c$

33. Let $I_n=\int\limits_{-\pi}^{\pi}\dfrac{\sin nx}{\sin x}dx$, $n \in N$ then
(a) $I_{n+2} = I_n$ (b) $\sum\limits_{m=1}^{20} I_{2m+1}=20\pi$
(c) $I_{2m} = 0$ where $m = 1, 2, 3, \ldots$..
(d) $I_{n+1} = I_n$

34. Let $f(x)=\begin{cases}2x-4 & x\leq 2\\ -x^2+\dfrac{k^3(k-1)^2}{k^2-k-2}+4, & x>2\end{cases}$.
$f(x)$ attains local maximum at $x = 2$ if k lies in
(a) (0, 1) (b) (3, ∞)
(c) (–∞, –1) (d) (1, 2)

35. $\sin^{-1}(x^2 + 2x + 2) + \tan^{-1}(x^2 - 3x - k^2) > \pi/2$ for $k \in$
(a) (–1, 0) (b) (0, 1) (c) (1, 2) (d) (0, 2)

36. If a right angled ΔABC of maximum area is inscribed within a circle of radius R, then (Δ represents area of triangle ABC and r, r_1, r_2, r_3 represent inradius and exradii, and s is the semi perimeter of ΔABC, then
(a) $\Delta = R^2$ (b) $\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}=\dfrac{\sqrt{2}+1}{R}$
(c) $r=(\sqrt{2}-1)R$ (d) $s=(1+\sqrt{2})R$

37. The values of a for which
$\dfrac{x^3-6x^2+11x-6}{x^3+x^2-10x+8}+\dfrac{a}{30}=0$ does not have a real solution is
(a) –10 (b) 12 (c) 5 (d) –30

38. Which of the following functions will not have absolute minimum value?
(a) $\cot(\sin x)$ (b) $\tan(\log x)$
(c) $x^{2005} - x^{1947} + 1$ (d) $x^{2006} + x^{1947} + 1$

39. If a_1, a_2, a_3, ..., a_n is sequence of +ve numbers which are in A.P. with common difference 'd' and $a_1 + a_4 + a_7 + \ldots + a_{16} = 147$, then
(a) $a_1 + a_6 + a_{11} + a_{16} = 98$
(b) $a_1 + a_{16} = 49$
(c) $a_1 + a_4 + a_7 + \ldots + a_{16} = 6a_1 + 45d$
(d) Maximum value of $a_1a_2 \ldots a_{16}$ is $\left(\dfrac{49}{2}\right)^{16}$

SECTION-III

(Comprehension Type)

This section contains paragraphs. Based upon each paragraph, multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Paragraph for Question Nos. 40 to 42

A person walks $2\sqrt{2}$ units away from origin in south west direction ($S\ 45°W$) to reach A, then walks $\sqrt{2}$ units in south east direction ($S\ 45°E$) to reach B. From B, he travels 4 units horizontally towards east to reach C, then he travels along a circular path with centre at origin through an angle of $\dfrac{2\pi}{3}$ in anti clockwise direction to reach his destination D.

40. Position of B in argand plane is
(a) $\sqrt{2}\,e^{-i\frac{3\pi}{4}}$ (b) $\sqrt{2}\,(2+i)e^{-i3\pi/4}$
(c) $\sqrt{2}\,(1+2i)e^{-i3\pi/4}$ (d) $-3 + i$

41. Let the complex number z represent C in argand plane, then $\arg(z)$ =

(a) $-\pi/6$ (b) $\pi/4$ (c) $-\pi/4$ (d) $\pi/3$

42. Position of D in argand plane is (ω is an imaginary cube root of unity)

(a) $(3 + i)\omega$ (b) $-(1 + i)\omega^2$

(c) $3(1 - i)\omega$ (d) $(1 - 3i)\omega$

Paragraph for Question Nos. 43 to 45

Let the curves $S_1 : y = x^2$, $S_2 : y = -x^2$, $S_3 : y^2 = 4x - 3$

43. Area bounded by the curves S_1, S_2, S_3 is

(a) $\frac{4}{3}$ sq.u (b) $\frac{8}{3}$ sq.u (c) $\frac{1}{6}$ sq.u (d) $\frac{1}{3}$ sq.u

44. Area bounded by the curves S_1, S_3 and the line $x = 3$ is

(a) $\frac{13}{3}$ sq.u (b) $\frac{5}{4}$ sq.u

(c) $\frac{8}{3}$ sq.u (d) $\frac{7}{4}$ sq.u

45. Area bounded by the curve S_3, $y \le -1$ and the line $x = 3$ is

(a) $\frac{7}{3}$ sq.u (b) $\frac{11}{3}$ sq.u (c) $\frac{9}{2}$ sq.u (d) $\frac{13}{4}$ sq.u

Paragraph for Question Nos. 46 to 48

Consider the planes,

$S_1 : 2x - y + z = 5$, $S_2 : x + 2y - z = 4$

having normals N_1 and N_2 respectively. $P(2, -1, 0)$ and $Q(1, 1, -1)$ are points on S_1 and S_2 respectively.

46. A vector of magnitude $\sqrt{140}$ units and lies along the line of intersection of S_1 and S_2 is

(a) $2(5\hat{i}+3\hat{j}-\hat{k})$ (b) $2(\hat{i}+3\hat{j}+5\hat{k})$

(c) $2\hat{i}-6\hat{j}-10\hat{k}$ (d) $2(3\hat{i}-\hat{j}+5\hat{k})$

47. The distance of the origin from the plane passing through the point (1, 1, 1) and whose normal is perpendicular to N_1 and N_2 is

(a) $\frac{9}{\sqrt{61}}$ (b) $\frac{11}{\sqrt{35}}$ (c) $\frac{10}{\sqrt{61}}$ (d) $\frac{7}{\sqrt{35}}$

48. Let L_1 be the line passing through P and parallel to N_1, L_2 be the line passing through Q and parallel to N_2. The shortest distance between L_1 and L_2 is

(a) $\frac{2}{\sqrt{35}}$ (b) $\frac{8}{\sqrt{35}}$ (c) $\frac{14}{\sqrt{35}}$ (d) $\frac{17}{\sqrt{35}}$

Paragraph for Question Nos. 49 to 51

A is a set containing n elements. A subset S_1 of A is chosen. The set A is reconstructed by replacing the elements of S_1. Again, a subset S_2 of A is chosen and again the set is reconstructed by replacing the elements of S_2. The number of ways of choosing S_1 or S_2 where

49. S_1 and S_2 have one element in common is

(a) 3^{n-1} (b) $n \cdot 3^{n-1}$ (c) 2^{n-1} (d) n

50. $S_1 \cup S_2 = A$ is

(a) 3^n (b) $n \cdot 3^n$ (c) 4^n (d) 4^{n-1}

51. S_1 is a subset of S_2 is

(a) 4^{n-1} (b) 3^{n+1} (c) 4^n (d) 3^n

Paragraph for Question Nos. 52 to 54

Let each of the circles

$S_1 \equiv x^2 + y^2 + 4y - 1 = 0$

$S_2 \equiv x^2 + y^2 + 6x + y + 8 = 0$

$S_3 \equiv x^2 + y^2 - 4x - 4y - 37 = 0$

touches the other two. Let P_1, P_2, P_3 be the point of contact of S_1 and S_2, S_2 and S_3, S_3 and S_1 respectively. Let T be the point of concurrence of the tangents at P_1, P_2, P_3 to the circles. C_1, C_2, C_3 are the centres of S_1, S_2, S_3 respectively.

52. P_2 and P_3 are reflections of each other in the line

(a) $y = x$ (b) $y = x + 1$

(c) $2x - y + 3 = 0$ (d) $2x + y + 9 = 0$

53. The area of the quadrilateral $TP_2C_3P_3$ is

(a) 11 sq. units (b) 25 sq.units

(c) 15 sq. units (d) 9 sq. units

54. The ratio of the area of $\frac{\Delta P_1P_2P_3}{\Delta C_1C_2C_3}$ is

(a) 5 : 2 (b) 5 : 3 (c) 3 : 2 (d) 2 : 5

Paragraph for Question Nos. 55 to 57

Whenever we have to find the sum of finite or infinite series of the form

$a_0 \cos\alpha + a_1 \cos(\alpha + \beta) + a_2 \cos(\alpha + 2\beta) + \ldots$

or, $a_0 \sin\alpha + a_1 \sin(\alpha + \beta) + a_2 \sin(\alpha + 2\beta) + \ldots$

then we will use following method

Step-I : If the series whose sum is to be found in cosine, let this series be denoted by C. Then write another corresponding auxiliary series in sines and denote it by S and vice versa.

Step-II : Find $C + iS$, use $e^{i\theta} = \cos\theta + i \sin\theta$ and simplify.

$C + iS$ series thus obtained, converts it to some standard series whose sum can be easily calculated.

Finally we convert this sum in $A + iB$.

Step-III : Now equate real and imaginary parts from both sides to get the required result.

55. The sum of the series

$$\cos\alpha + x\cos(\alpha+\beta) + \frac{x^2}{2!}\cos(\alpha+2\beta) + \ldots\infty \text{ is}$$

(a) $e^{x\cos\beta}\cdot\cos(\alpha + x\sin\beta)$

(b) $e^{x\cos\beta}\sin(\alpha + x\sin\beta)$

(c) $e^{x\sin\beta}\cdot\cos(\alpha + x\sin\beta)$

(d) $e^{x\sin\beta}\cdot\sin(\alpha + x\sin\beta)$

56. The sum of the series $\sin\theta - \frac{\sin 2\theta}{2!} + \frac{\sin 3\theta}{3!} - \ldots\infty$ is

(a) $e^{-\cos\theta}\cdot\sin(\sin\theta)$ (b) $e^{-\sin\theta}\cdot\cos(\sin\theta)$

(c) $e^{-\cos\theta}\cdot\cos(\sin\theta)$ (d) $e^{-\sin\theta}\cdot\sin(\sin\theta)$

57. $\cos\frac{\pi}{3} + \frac{1}{2}\cos\frac{2\pi}{3} + \frac{1}{3}\cos\frac{3\pi}{3} + \ldots\infty$ is equal to

(a) $\pi/3$ (b) 0 (c) 1 (d) $e^{i\pi/3}$

Paragraph for Question Nos. 58 to 60

If n distinct objects are distributed randomly into n distinct boxes, what is the probability that

58. No box is empty

(a) $\frac{\lfloor n-1}{n^n}$ (b) $\frac{\lfloor n-1}{\lfloor 2n^n}$ (c) $\frac{\lfloor n}{n^n}$ (d) $\frac{\lfloor 2\lfloor n-1}{n^n}$

59. Exactly one box empty

(a) $\frac{\lfloor n\ {}^nC_2}{n^{n-1}}$ (b) $\frac{\lfloor n\ {}^nC_2}{n^n}$

(c) $\frac{n\lfloor n}{n^n}$ (d) $\frac{\lfloor n}{n^n}$

60. A particular box get exactly r objects

(a) $\frac{{}^nC_r(n-1)^{n-r-1}}{n^n}$ (b) $\frac{{}^nC_r(n-1)^{n-r+1}}{n^n}$

(c) $\frac{{}^nC_r(n-1)^{n-r}}{n^{n-1}}$ (d) $\frac{{}^nC_r(n-1)^{n-r}}{n^n}$

Paragraph for Question Nos. 61 to 63

Let z_1 and z_2 be complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$. Also suppose that roots α and β of $t^2 + z_1 t + z_2 + m = 0$ for some complex number m satisfying $|\alpha-\beta| = 2\sqrt{7}$

61. The complex number m lies on

(a) a square with side 7 and centre (4, 5)

(b) a circle with radius 7 and centre (4, 5)

(c) a circle with radius 7 and centre (–4, 5)

(d) a square with side 7 and centre (–4, 5)

62. The greatest value of $|m|$ is

(a) $5+\sqrt{21}$ (b) $5+\sqrt{23}$

(c) $7+\sqrt{43}$ (d) $7+\sqrt{41}$

63. The least value of $|m|$ is

(a) $7-\sqrt{41}$ (b) $7-\sqrt{43}$

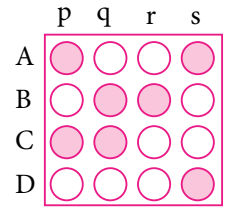
(c) $5-\sqrt{23}$ (d) $5+\sqrt{21}$

SECTION-IV

(Matrix – Match Type)

This section contains questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A – p, s, B – q, r, C – p, q and D – s, then the correctly bubbled 4 × 4 matrix should be as follows:



64. Match the following:

Column I		Column II	
(A)	The greatest value of $2x^3 - 3x^2 - 12x + 1$, where $2x^2 - x - 10 \le 0$ is	(p)	1
(B)	Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$, if $A^{-1} = aA^2 + bA + cI$, where $a, b, c \in R$, then the value of $2a + b + c$ is	(q)	3
(C)	If $\binom{n}{r} = {}^nC_r$ and $\sum_{r=0}^{30}\binom{30}{r}\binom{50}{20+r} = \binom{N}{K}$, then $\frac{K}{10}$ can take the value(s)	(r)	5
(D)	If $\tan\theta = \cot\phi$, then $\frac{2(\theta+\phi)}{\pi}$ can take the value(s)	(s)	8

65. Match column I with column II.

Column I		Column II	
(A)	If $a = 7$, $b = 7\sqrt{3}$ in ΔABC right angled at C, then $\angle A$ is equal to	(p)	$\pi/2$
(B)	The angle between the tangents drawn from the point (0, 2) to the curve $y^2 - 4x - 4y + 8 = 0$, is	(q)	$3\pi/4$
(C)	The angle between the tangents drawn from the point (2, 3) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, is	(r)	$\pi/6$
(D)	If $\sec\theta \cos 5\theta = -1$, where $0 < \theta < \pi$, then θ is equal to	(s)	$\pi/4$

66. Match the following:

Column I		Column II	
(A)	The maximum value of $\sin(\cos x) + \cos(\sin x)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is	(p)	cos(cos1)
(B)	The minimum value of $\sin(\cos x) + \cos(\sin x)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is	(q)	1 + cos1
(C)	The maximum value of $\cos(\cos(\sin x))$ is	(r)	cos1
(D)	The minimum value of $\cos(\cos(\sin x))$ is	(s)	1 + sin1

67. Match the following :

Column I		Column II	
(A)	If $f(x)=\sqrt{4^x+8^{\frac{2}{3}(x-1)}-72-4^{x-\frac{3}{2}}}$ is defined $\forall\, x \geq a$, then $f(a)$ =	(p)	0
(B)	If $2f(x^2)+3f\left(\frac{1}{x^2}\right)=x^2-1$, $(x \neq 0)$ then $f(1)$ =	(q)	4
(C)	$f(x)=[x]+\sum_{r=1}^{398}\frac{\{x+r\}}{398}$, where $[\cdot]$ denotes the greatest integer function and $\{\cdot\}$ denotes the fractional part of x, then $f(3)$ =	(r)	3
(D)	The number of points of discontinuity of $f(x)=[\tan x[\cot x]]$; $x \in \left[\frac{\pi}{12}, \frac{\pi}{2}\right)$ where [.] denotes the greatest integer function.	(s)	2

68. Match the following :

Column I		Column II	
(A)	If ω is a cube roots of unity, then $\left\lvert\omega+\omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\ldots\infty\right)}\right\rvert$ is equal to	(p)	0
(B)	$\int_{-1}^{1}\left\{\sin^{-1}\left[x+\frac{3}{4}\right]\right\}dx = K\left(\frac{\pi}{4}\right)$ then K is equal to (where $[\cdot]$ denotes the greatest integer function)	(q)	1
(C)	$\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{n+2}+\frac{1}{n+4}+\ldots+\frac{1}{3n}\right] = \frac{1}{2}\ln k$, then the value of k is	(r)	3
(D)	z_1 and z_2 are two complex number satisfying $\lvert z_1 + 1\rvert + \lvert z_1 - 1\rvert = 4$ and $\lvert z_2 - 2\rvert = 1$ then the maximum value of $\lvert z_1 - z_2\rvert$ is	(s)	5

SECTION-V

(Integer Answer Type Questions)

This section contains questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9.

69. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, then the value of $8\,\text{cosec}^6\alpha - 27\sec^6\alpha$ is

70. A line passing through (21, 30) and normal to the curve $y = 2\sqrt{x}$. If m is slope of the normal, then $m + 6$ =

71. The number of solutions of the equation $\sin^{-1}\frac{1+x^2}{2x}=\frac{\pi}{2}\sec(x-1)$ are

72. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}$ is perpendicular to plane of $\vec{b}$ and $\vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/3$, then $|\vec{a}+\vec{b}+\vec{c}|$ is

73. $f(x)=\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x}, & x<\dfrac{\pi}{2}\\ a, & x=\dfrac{\pi}{2}\\ \dfrac{b(1-\sin x)}{(\pi-2x)^2}, & x>\dfrac{\pi}{2}\end{cases}$

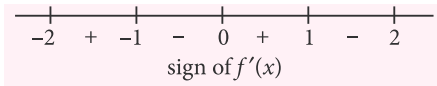
If $f(x)$ is continuous at $x = \pi/2$, then $\dfrac{b}{a}=$

74. If $I=\int\limits_{-\pi/2}^{2\pi}\sin^{-1}(\sin x)\,dx$, then $-\dfrac{16}{\pi^2}I=$

ANSWERS

1. (b): $f'(x)=\begin{cases}3(x+1)^2 & -2<x<-1\\ \dfrac{2}{3}\times(x^{-1/3}) & -1<x<1-\{0\}\\ -2(x-1) & 1<x<2\end{cases}$

$f'(x)$ does not exist at $x = -1, 0, 1$



2. (d): $\lim\limits_{x\to0^+}(\sqrt{\tan x})^{\sqrt{x}}+\lim\limits_{x\to0^+}(\cos x)^{-1/x}$

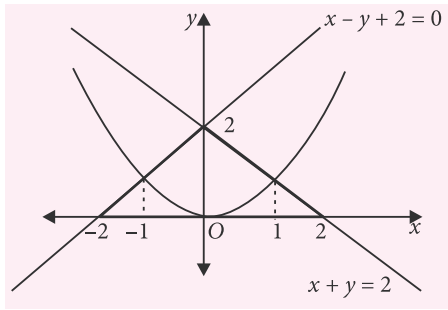
$e^{\lim\limits_{x\to0^+}\frac{\log_e(\sqrt{\tan x})}{\frac{1}{\sqrt{x}}}\left(\frac{\infty}{\infty}\right)}=e^0=1$

$\lim\limits_{x\to0^+}(\cos x)^{-1/x}=1$ as $0<\cos x<1$

3. (c): Since, given vectors are coplanar,

$$\begin{vmatrix}1 & p^2 & p^2\\ p^2 & 1 & p^4\\ p^4 & p^4 & 1\end{vmatrix}=0$$

4. (b):



(a, a^2) lies on $y = x^2$

$\Rightarrow\ a - a^2 + 2 = 0\ \Rightarrow\ a = -1, 2$

and $a + a^2 = 2\ \Rightarrow\ a = 1, -2$

5. (b): Let 'A' be the event of guessing the correct answer.

'B' be the event of copying the correct answer.

'C' be the event of knowing the correct answer.

'D' be the event that his answer is correct

$P(A) = 1/3$; $P(B) = 1/6$; $P(C) = 1/2$; $P(D|B) = 1/8$

$P(D|A) = 1/4$; $P(D|C) = 1$

$$P(C/D)=\frac{P(C)\cdot P(D/C)}{P(A)P(D/A)+P(B)(D/B)+P(C)(D/C)}=\frac{24}{29}$$

6. (c): $\because$ P is nearest to $3x + 4y - 10 = 0$

$\therefore\ \dfrac{2a}{e}=\dfrac{|3(6)+4(8)-10|}{\sqrt{36+64}}=4\Rightarrow a=2e,$

$\Rightarrow\ ae-\dfrac{a}{e}=8\Rightarrow e=\sqrt{5}, a=2\sqrt{5}$

$CP = ae = 10$

7. (d): $x^3 = 1\ \Rightarrow\ x = \omega, \omega^2$

$x^r+x^{2r}=\begin{cases}2, \text{if } r \text{ is a multiple of } 3\\ -1, \text{ if } r \text{ is not a multiple of } 3\end{cases}$

8. (d): 1) $ad = 1, bc = 4\ \Rightarrow\ (a = 1, d = 1, b = 2, c = 2)$

2) $ad = 1, bc = 2\ \Rightarrow\ (a = 1, d = 1, b = 1, c = 2);$ $(a = 1, d = 1, b = 2, c = 1)$

3) $ad = 2, bc = 1\ \Rightarrow\ (a = 1, d = 2, b = 1, c = 1);$ $(a = 2, d = 1, b = 1, c = 1)$

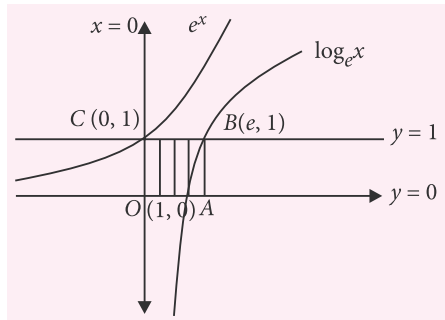
4) $ad = 2, bc = 4\ \Rightarrow\ (a = 1, d = 2, b = 2, c = 2);$ $(a = 2, d = 1, b = 2, c = 2)$

5) $ad = 4, bc = 1\ \Rightarrow\ (a = 2, d = 2, b = 1, c = 1)$

6) $ad = 4, bc = 2\ \Rightarrow\ (a = 2, d = 2, b = 1, c = 2);$ $(a = 2, d = 2, b = 2, c = 1)$

$\Rightarrow$ required probability $=\dfrac{10}{16}=\dfrac{5}{8}$

9. (d):



Area = Area of rectangle $OABC$ $-\int\limits_1^e\log_e x\,dx$

$= e - 1$ sq. units

10. (d): $E_1=(A\cap\bar{B})\cup(\bar{A}\cap B),\ E_2=(B\cap\bar{C})\cup(\bar{B}\cap C),$

$E_3=(C\cap\bar{A})\cup(\bar{C}\cap A), E_4=A\cap B\cap C, E_5=A\cup B\cup C$

11. (a): $M(x)=\int\dfrac{e^x(e^{2x}-2)}{e^{4x}+8e^{2x}+4}dx$ (Put $e^x = t$)

$$\Rightarrow \int \frac{(t^2-2)\,dt}{t^4+8t^2+4} = \frac{1}{2}\tan^{-1}\left(\frac{t+2/t}{2}\right)+c$$
$$= \frac{1}{2}\tan^{-1}\left(\frac{e^x+2e^{-x}}{2}\right)+c$$

12. (c) : $\hat{n} = \dfrac{\hat{u}-\hat{v}}{|\hat{u}-\hat{v}|}$, $|\hat{u}-\hat{v}| = \sqrt{1+1-2(1)(1)\left(-\frac{1}{2}\right)} = \sqrt{3}$

13. (a) : At $x = 0$, $y = 2$, $y'(0) = 4$
Equation of normal is $x + 4y = 8$. $(\alpha, 0)$ lies on normal
$\Rightarrow \alpha = 8$

14. (c) : $f(x) = (x - a_1)(x - a_2)(x - a_3) + (a_1 - x) + (a_2 - x) + (a_3 - x)$
Now $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.
Again $f(a_1) = (a_2 - a_1) + (a_3 - a_1) > 0$ [$\because a_1 < a_2 < a_3$]
$\Rightarrow$ One root belongs to $(-\infty, a_1)$
Also, $f(a_3) = (a_1 - a_3) + (a_2 - a_3) < 0$
$\Rightarrow$ One root belongs to (a_1, a_3)
So $f(x) = 0$ has three distinct real roots.

15. (c) : $F'(x) = f(x)$
$F(x) = x(1+\sqrt{x}) = x + x^{3/2}$
$\therefore \quad F'(x) = f(x) = 1 + \dfrac{3}{2}\sqrt{x}$
$\therefore \quad f(4) = 4$

16. (a) : $f(x) = \int_0^x (1+t^3)^{-1/2}\,dt$
i.e., $f[g(x)] = \int_0^{g(x)} (1+t^3)^{-1/2}\,dt$
i.e., $x = \int_0^{g(x)} (1+t^3)^{-1/2}\,dt$ [$\because g$ is inverse of $f \Rightarrow f[g(x)] = x$]
Differentiating with respect to x, we have
$1 = (1 + g^3)^{-1/2} g'$
i.e., $(g')^2 = 1 + g^3$
Differentiating again with respect to x, we have
$2g'g'' = 3g^2g'$ gives $\dfrac{g''}{g^2} = \dfrac{3}{2}$

17. (c) : Taking $x = y = 1$, we get
$f(1) f(1) - f(1) = 2$
$\Rightarrow f^2(1) - f(1) - 2 = 0 \Rightarrow (f(1) - 2)(f(1) + 1) = 0$
$\Rightarrow f(1) = 2$ (as $f(1) > 0$)
Taking $y = 1$, we get
$f(x) \cdot f(1) - f(x) = x + 1$
$\Rightarrow f(x) = x + 1 \Rightarrow f^{-1}(x) = x - 1$
$\therefore \quad f(x) \cdot f^{-1}(x) = x^2 - 1$

18. (c) : $V = 1 + a^3 - a$, $\dfrac{dV}{da} = 3a^2 - 1$, $\dfrac{d^2V}{da^2} = 6a$
$\Rightarrow$ V has a minimum at $a = \dfrac{1}{\sqrt{3}}$

19. (c) : Let α, β be roots of $x^2 - bx + c = 0$,
Then $\alpha + \beta = b$
$\Rightarrow$ one of the roots is '2' (Since α, β are primes and b is odd positive integer)
$\therefore \quad f(2) = 0 \Rightarrow 2b - c = 4$ and $b + c = 35$
$\therefore \quad b = 13$, $c = 22$
Minimum value $= f\left(\dfrac{13}{2}\right) = -\dfrac{81}{4}$

20. (b): We have $A^T = -A$
$(A^4)^T = (A \cdot A \cdot A \cdot A)^T = A^T A^T A^T A^T$
$\Rightarrow (-A)(-A)(-A)(-A) = (-1)^4 \cdot A^4 = A^4$

21. (d): $x^n + ax + b = (x - \alpha_1)(x - \alpha_2) \ldots (x - \alpha_n)$
Differentiate both sides w.r.t. x
$nx^{n-1} + a = (x - \alpha_2) \ldots (x - \alpha_n) + (x - \alpha_1)\left(\dfrac{d}{dx}(x-\alpha_2)\ldots(x-\alpha_n)\right)$
Put $x = \alpha_1$;
$n\alpha_1^{n-1} + a = (\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3) \ldots (\alpha_1 - \alpha_n)$

22. (c) : Both the circles have radius = 5 and they intersect each other, therefore their common tangent is parallel to the line joining their centres.
Equation of the line joining their centre is
$7x - 5y + 1 = 0$.
$\therefore$ Equation of the common tangent is $7x - 5y = c$
$\therefore \quad \left|\dfrac{c+1}{\sqrt{74}}\right| = 5 \Rightarrow c = \pm 5\sqrt{74} - 1$
$\therefore$ equation is $7x - 5y + 1 \mp 5\sqrt{74} = 0$.

23. (a) : Since $x^2 + y^2 = 25$
$\Rightarrow x = 5\cos\theta$ and $y = 5\sin\theta$
So, therefore, $\log_5(3x + 4y) = \log_5(15\cos\theta + 20\sin\theta)$
$\Rightarrow \{\log_5(3x + 4y)\}_{\max} = 2$

24. (c) : $X = AB + BA \Rightarrow X^T = X$
and $Y = AB - BA \Rightarrow Y^T = -Y$
Now, $(XY)^T = Y^T \times X^T = -YX$

25. (c) : $\dfrac{2y}{(1-y^2)^2} \cdot \dfrac{dy}{dx} + \dfrac{y^2}{1-y^2} \cdot \dfrac{1}{x} = \dfrac{1}{x^3}$
Put $\dfrac{y^2}{1-y^2} = t \Rightarrow \dfrac{2y}{(1-y^2)^2}\dfrac{dy}{dx} = \dfrac{dt}{dx}$
$\Rightarrow \dfrac{dt}{dx} + \dfrac{t}{x} = \dfrac{1}{x^3} \Rightarrow t \cdot x = \int \dfrac{1}{x^2}\,dx + c$
$\Rightarrow x^2y^2 = (cx - 1)(1 - y^2)$

26. (c) : Family of lines passes through focus hence latus rectum will makes shortest intercept.

27. (a) : $2\sin^2 x + \frac{2\sin x \cos x}{2} = n$

$\sin 2x - 2\cos 2x = 2n - 2$

$-\sqrt{5} \le 2n-2 \le \sqrt{5} \Rightarrow 1-\frac{\sqrt{5}}{2} \le n \le 1+\frac{\sqrt{5}}{2}$

28. (c) : $\sin^2\left(\frac{x}{2}\right) = \frac{x}{(2n+1)\pi}$

The graph of $\sin^2(x/2)$ will be above the x-axis and will be meeting the x-axis at 0, 2π, 4π, ... etc. It will attain maximum values at odd multiples of π *i.e.* at π, 3π, ... $(2n + 1)\pi$. The last point after which graph of $y = \frac{x}{(2n+1)\pi}$ will stop cutting is $(2n + 1)\pi$.

Total intersection = $2(n + 1)$.

29. (c) : If the tangents at P and Q intersect at T, then axis of parabola is parallel to TR, where R is the mid point of P and Q. So, slope of the axis is 1.

$\therefore$ slope of the directrix = – 1.

30. (c) : $\log_e\left(\lim_{x\to x}[f(x)] + x^{2)^{\frac{1}{\{f(x)\}}}}\right)$ ...(i)

$\lim_{x\to 0}[f(x)] = \lim_{x\to 0}\left[\frac{\tan x}{x}\right] = 1$

$\lim_{x\to 0}([f(x)] + x^2)^{\frac{1}{\{f(x)\}}} = \lim_{x\to 0}(1+x^2)^{\frac{1}{\{f(x)\}}}$ (1^∞ form)

Again, $f(x) = \frac{\tan x}{x} = \frac{x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots}{x}$

$= 1 + \frac{x^2}{3} + \frac{2}{15}x^4 + \ldots$

$\{f(x)\} = \frac{x^2}{3} + \frac{2}{15}x^4 + \ldots$

(i) becomes,

$\log_e\left(e^{\lim_{x\to 0} x^2 \times \frac{1}{\{f(x)\}}}\right) = \lim_{x\to 0}\frac{x^2}{\frac{x^2}{3} + \frac{2}{15}x^4 + \ldots} = 3$

31. (b): $a_n = \frac{(\log_e 10)^n}{n!}\sum_{k=0}^{n}\frac{n!}{k!(n-k)!}$

$= \frac{(\log_e 10)^n}{n!}[2^n] = \frac{(2\log_e 10)^n}{n!}$

Thus, $a_0 + a_1 + a_2 + \ldots$ upto infinity is

$= \sum_{n=0}^{\infty}\frac{(2\log_e 10)^n}{n!} = e^{2\log_e 10} = 10^2$

32. (b, c) : $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \quad a = b\cos C \pm \sqrt{c^2 - (b\sin C)^2}$

33. (a, c) : $I_n = 2\int_0^{\pi}\frac{\sin nx}{\sin x}dx$

$\left(\because f(x) = \frac{\sin nx}{\sin x}\text{ is an even function}\right)$

$\Rightarrow \quad I_{n+2} - I_n = 0$

34. (a, c, d) : When $f(x)$ is continuous at $x = 2$

$f'(x)$ does not exist at $x = 2$ and $f'(x)$ changes sign from + to –

$\Rightarrow$ $f(x)$ attains max. at $x = 2$ if

$\frac{k^3(k-1)^2}{k^2-k-2} = 0 \Rightarrow k = 0, 1$

When $f(x)$ is discontinuous at $x = 2$, $f'(x)$ changes its sign from + to –. $f(x)$ will attain maximum if $\lim_{x\to 2^+} f(x) < f(2)$ as

$\lim_{x\to 2^-} f(x) = f(2)$ *i.e.* if $k \in (-\infty, -1) \cup (0, 1) \cup (1, 2)$

$\Rightarrow$ $k \in (-\infty, -1) \cup [0, 2)$

35. (a, b, c, d) : It is satisfied for only $x = -1$

$\frac{\pi}{2} + \tan^{-1}(x^2 - 3x - k^2) > \frac{\pi}{2}$ for $x = -1$

$\Rightarrow \quad k^2 - 4 < 0$

36. (a, b, c, d) : For a right angled triangle inscribed in a circle of radius R, the length of the hypotenuse is $2R$. Then area is maximum when it is an isosceles triangle with each side $= \sqrt{2}R$

$\therefore \quad s = \frac{1}{2}(2\sqrt{2} + 2)R = (\sqrt{2} + 1)R$

$\Delta = \frac{1}{2}\sqrt{2}R \cdot \sqrt{2}R = R^2$

$r = \frac{\Delta}{s} = \frac{R^2}{(\sqrt{2}+1)R} \Rightarrow r = (\sqrt{2} - 1)R$

$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r} = \frac{1}{(\sqrt{2}-1)R} = \frac{\sqrt{2}+1}{R}$

37. (b, c, d) : $\dfrac{x^3-6x^2+11x-6}{x^3+x^2-10x+8}=\dfrac{(x-1)(x-2)(x-3)}{(x-1)(x-2)(x+4)}$

$\therefore \quad x \neq 1, 2, -4$ then $f(x)=\dfrac{x-3}{x+4}$

Range of $f(x)=R-\left\{1,-\dfrac{2}{5},-\dfrac{1}{6}\right\}$

So equation does not have a solution if $\dfrac{a}{30}=-1,\dfrac{2}{5},\dfrac{1}{6}$

$\Rightarrow \quad a = -30, 12, 5$

38. (a, b, c) : Even degree polynomial with leading coefficient +ve will have absolute minimum.

39. (a, b, c, d) : $a_1 + a_4 + a_7 + ... + a_{16} = 147$

$\Rightarrow \quad 3(a_1 + a_{16}) = 147 \quad \Rightarrow \quad a_1 + a_{16} = 49$

Again $a_1 + a_4 + a_7 + a_{10} + ... + a_{16}$

$= a_1 + a_1 + 3d + a_1 + 6d + ... + a_1 + 15d$

$= 6a_1 + 45d = 147 \quad \Rightarrow \quad 2a_1 + 15d = 49$

$a_1 + a_6 + a_{11} + a_{16} = a_1 + a_1 + 5d + a_1 + 10d + a_1 + 15d$

$= 4a_1 + 30d = 2(2a_1 + 15d) = 2(49) = 98$

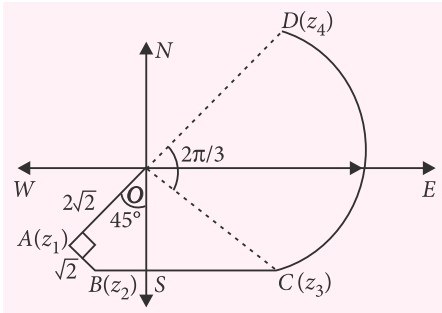
Now using A.M. $\geq$ G.M.

$$\frac{a_1+a_2+...+a_{16}}{16} \geq (a_1a_2a_3 ... a_{16})^{\frac{1}{16}}$$

$$\Rightarrow \quad \frac{8(a_1+a_{16})}{16} \geq (a_1a_2a_3 ... a_{16})^{1/16}$$

$$\Rightarrow \quad \left(\frac{49}{2}\right)^{16} \geq a_1a_2a_3 ... a_{16}$$

40. (b) **41. (c)** **42. (c)**



$z_1 = 2\sqrt{2}\ e^{-i3\pi/4} = -2 - 2i$

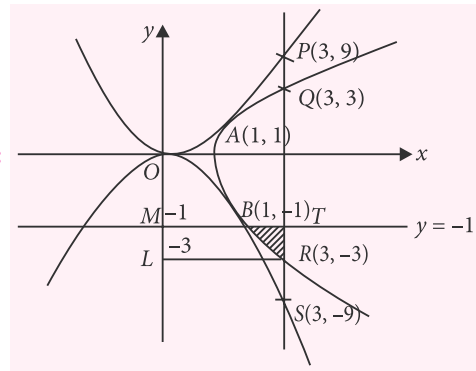
$\dfrac{z_2-(-2-2i)}{0-(-2-2i)}=\dfrac{1}{2}\times e^{-i\pi/2}$ (rotation at A)

$\Rightarrow \quad z_2 = -(1 + i)(2 + i) = -1 - 3i$

$\therefore \quad z_3 = 3 - 3i$

$\dfrac{z_4-0}{3-3i-0}=e^{i2\pi/3} \Rightarrow z_4 = 3(1-i)\omega$ (rotation at O)

43. (d):



Area $OAB = 2\int_0^1\left(\dfrac{y^2+3}{4}-\sqrt{y}\right)dy=\dfrac{1}{3}$

44. (a) : Area $APQ=\int_1^3(x^2-\sqrt{(4x-3)})dx$

45. (a) : Area BTR = Area of rectangle $LMTR$ – Area $LMBR$

$$=6-\int_{-3}^{-1}\frac{y^2+3}{4}dy$$

46. (c) **47. (d)** **48. (a)**

49. (b): Required number of ways = ${}^nC_1\cdot(3)^{n-1}$

50. (a) : Each element $\in S_1 \cup S_2$ in 3 ways

51. (d): If S_2 has r elements then S_1 and S_2 can be choosen in ${}^nC_r\ 2^r$ ways.

52. (a) **53. (c)** **54. (d)**

55. (a) : Let,

$$C=\cos\alpha+x\cos(\alpha+\beta)+\frac{x^2}{2!}\cos(\alpha+2\beta)+...\infty$$

and $S=\sin\alpha+x\sin(\alpha+\beta)+\dfrac{x^2}{2!}\sin(\alpha+2\beta)+...\infty$

$\therefore \quad C + iS$

$= (\cos\alpha + i\sin\alpha) + x(\cos(\alpha + \beta) + i\sin(\alpha + \beta))$

$$+\frac{x^2}{2!}\left(\cos(\alpha+2\beta)+i\sin(\alpha+2\beta)\right)+...\infty$$

$$=e^{i\alpha}+x\cdot e^{i(\alpha+\beta)}+\frac{x^2}{2!}e^{i(\alpha+2\beta)}+...\infty$$

$$=e^{i\alpha}\left[1+xe^{i\beta}+\frac{x^2}{2!}e^{i2\beta}+...\infty\right]$$

$$=e^{i\alpha}e^{x\cdot e^{i\beta}}=e^{i\alpha}\cdot e^{x(\cos\beta+i\sin\beta)}$$

$$=e^{x\cos\beta}\left[\cos(\alpha+x\sin\beta)+i\sin(\alpha+x\sin\beta)\right]$$

Equating real parts on both sides, we get

$C = e^{x\cos\beta}\cos(\alpha + x\sin\beta)$

56. (a): Let $C=\cos\theta-\frac{\cos 2\theta}{2!}+\frac{\cos 3\theta}{3!}-\ldots\infty$

$$S=\sin\theta-\frac{\sin 2\theta}{2!}+\frac{\sin 3\theta}{3!}-\ldots\infty$$

Then $C+iS=e^{i\theta}-\frac{1}{2!}e^{i2\theta}+\frac{1}{3!}e^{i3\theta}-\ldots\infty$

$\Rightarrow\ C+iS=1-e^{-e^{i\theta}}$

$\Rightarrow\ C+iS=1-e^{-\cos\theta}[\cos(\sin\theta)-i\sin(\sin\theta)]$

Now, equating imaginary parts on both sides, we get

$S = e^{-\cos\theta}\sin(\sin\theta)$

57. (b): Let $C=\cos\frac{\pi}{3}+\frac{1}{2}\cos\left(\frac{2\pi}{3}\right)+\frac{1}{3}\cos\left(\frac{3\pi}{3}\right)+\ldots\infty$

And $S=\sin\frac{\pi}{3}+\frac{1}{2}\sin\left(\frac{2\pi}{3}\right)+\frac{1}{3}\sin\left(\frac{3\pi}{3}\right)+\ldots\infty$

$$C+iS=e^{i\pi/3}+\frac{1}{2}e^{i(2\pi/3)}+\frac{1}{3}e^{i(3\pi/3)}+\ldots\infty$$

$$=-\log_e(1-e^{i\pi/3})=-\log_e\left[1-\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\right]$$

$$=-\log_e\left[\frac{1}{2}-i\frac{\sqrt{3}}{2}\right]=0+i\frac{\pi}{3}$$

Equating real parts from both sides, we get $C = 0$

58. (c): No box empty. The no. of favorable ways = $n!$

59. (b): Exactly one box empty, then no. of favorable ways $= {}^nC_1\cdot{}^{n-1}C_1\ {}^nC_2\lfloor n-2$

60. (d): A particular box get exactly r objects

$= {}^nC_r\ (n-1)^{n-r}$

61. (b) **62. (d)** **63. (a)**

$\alpha + \beta = -z_1$; $\alpha\beta = z_2 + m$

$(\alpha - \beta)^2 = z_1^2 - 4z_2 - 4m$

$|(\alpha - \beta)|^2 = 16 + 20i - 4m$

$|\alpha - \beta|^2 = |16 + 20i - 4m|$

$\because\ |\alpha-\beta|=2\sqrt{7}\ |\Rightarrow m-5i-4|=7\ \Rightarrow\ |m-(4+5i)|=7$

$\therefore$ m lies on a circle having centre (4, 5) and radius 7

64. A → s ; B → p ; C → q, r ; D → p, q, r

(A) $f'(x) = 0\ \Rightarrow\ x = -1, 2$

$f(-1) = 8, f(-2) = -3, f(2) = -19,\ f\left(\frac{5}{2}\right)=\frac{-33}{2}$

(B) A satisfies $|A - \lambda I| = 0\ \Rightarrow A^3 - 2A^2 + 2A - 2I = 0$

(C) Coefficient of x^{30} in $(1 + x)^{30} \times (1 + x)^{50}$

$= {}^{80}C_{30} = {}^{80}C_{50}$

$\Rightarrow\ \frac{k}{10}=3, 5$

(D) $\tan\theta = \cot\phi\ \Rightarrow\ \cos(\theta + \phi) = 0$

$\Rightarrow\ \theta + \phi = (2n + 1)\pi/2$

$\Rightarrow\ \frac{2(\theta+\phi)}{\pi}=2n+1,\ n\in I$

65 A → r ; B → p ; C → p ; D → q, r, s

(A) $\tan A=\frac{7}{7\sqrt{3}}=\frac{1}{\sqrt{3}}$

$\angle A = \pi/6$.

(B) (0, 2) is the point on the directrix of the parabola $y^2 - 4x - 4y + 8 = 0$

$\therefore$ angle $= \pi/2$

(C) (2, 3) is the point on the director circle of the ellipse $\frac{x^2}{9}+\frac{x^2}{4}=1$

$\therefore$ angle $= \pi/2$.

(D) $\cos5\theta = -\cos\theta = \cos(\pi - \theta)$ $\qquad (\cos\theta \neq 0)$

$\Rightarrow\ 5\theta = 2n\pi \pm (\pi - \theta)$

$\Rightarrow\ \theta=\frac{n\pi}{3}+\frac{\pi}{6}$ or $\theta=\frac{n\pi}{2}-\frac{\pi}{4}$

$\Rightarrow\ \theta=\frac{\pi}{6},\frac{\pi}{4},\frac{3\pi}{4}$ and $\frac{5\pi}{6}$ between 0 to π.

66. A → s; B → r; C → p; D → r

Let $f(x) = \sin(\cos x) + \cos(\sin x)$

f is an even function. We can take $x\in\left[0,\frac{\pi}{2}\right]$. In $\left[0,\frac{\pi}{2}\right]$ $\sin x$ is increasing and $\cos x$ is decreasing.

Hence f is a decreasing function.

Therefore, maximum value of f is $f(0) = \sin 1 + 1$ and minimum value is $f(\pi/2) = 0 + \cos 1$.

Let $g(x) = \cos(\cos(\sin x))$. Obviously g is an even periodic function of period π. Hence g takes all of its values for $x\in\left[0,\frac{\pi}{2}\right]$.

It can be seen that g is an increasing function in $\left[0,\frac{\pi}{2}\right]$

So maximum value of $g = g(\pi/2) = \cos(\cos 1)$, and minimum value of $g = g(0) = \cos 1$.

67. A → r; B → p; C → r; D → r

(A) $4^x + 8^{\frac{2}{3}(x-1)} - 72 - 4^{x-\frac{3}{2}} \geq 0$

$\Rightarrow 2^{2x} + 2^{2x-2} - 72 - 2^{2x-3} \geq 0$

$\Rightarrow 2^{2x}\left[1 + \frac{1}{4} - \frac{1}{8}\right] \geq 72$

$\Rightarrow 2^{2x} \geq 64$ and $x \geq 3$

$\Rightarrow f(3) = 0$

(B) Put $x = 1$ to get $f(1)=0$

(C) $\sum_{r=1}^{398} \frac{\{x+r\}}{398} = \frac{398}{398}\{x\} = \{x\}$

$\therefore \quad f(x) = [x] + \{x\} = x \Rightarrow f(3) = 3$

(D) $f(x)$ is discontinuous when $\cot x \in$ integer

As $\frac{\pi}{12} \leq x < \frac{\pi}{2}$

$\therefore \quad 0 < \cot x \leq 2 + \sqrt{3}$

$\therefore$ Number of points of discontinuity = 3.

68. A → q; b → q; C → r; D → s

(A) $\frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \frac{27}{128} + \ldots \infty$

$= \frac{1}{2}\left[\left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^1 + \left(\frac{3}{4}\right)^2 + \ldots \infty\right] = \frac{1}{2}\left[\frac{1}{1-\frac{3}{4}}\right] = 2$

So therefore,

$\left|\omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\ldots\infty\right)}\right| = |\omega + \omega^2| = |-1| = 1$

(B) $\int_{-1}^{1} \left\{\sin^{-1}\left[x + \frac{3}{4}\right]\right\} dx = K\left(\frac{\pi}{4}\right)$

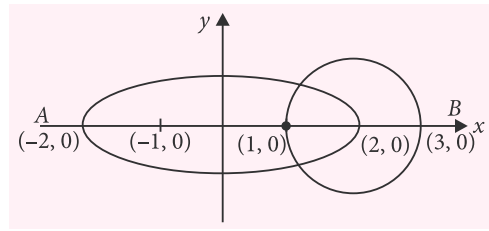
$\Rightarrow \int_{-1}^{-3/4} -\frac{\pi}{2}\,dx + \int_{-3/4}^{1/4} 0\,dx + \int_{1/4}^{1} \frac{\pi}{2}\,dx$

$\Rightarrow -\frac{\pi}{8} + \frac{\pi}{2} \times \frac{3}{4} \Rightarrow \frac{\pi}{4} = K\left(\frac{\pi}{4}\right) \Rightarrow K = 1$

(C) $\sum_{r=0}^{n} \frac{1}{n+2r} = \sum_{r=0}^{n} \frac{1}{n} \cdot \frac{1}{1+2\left(\frac{r}{n}\right)}$

$\int_0^1 \frac{dx}{1+2x} = \frac{[\ln(1+2x)]_0^1}{2} = \frac{1}{2}[\ln 3 - 0] = \frac{\ln 3}{2}$

(D) Locus of z_1 is an ellipse having foci at $(-1 + 0{\cdot}i)$ and $(1 + 0{\cdot}i)$



Length of the major axis = 4

Locus of z_2 is a circle having centre at (2, 0) and radius 1.

Max. value of $|z_1 - z_2| = AB = 5$

69. (0): From the given equation we can find that the value of $\tan^2\alpha = \frac{2}{3}$.

70. (1): Equation of the normal is

$y = mx - 2m - m^3$

If it pass through (21, 30), we have

$30 = 21m - 2m - m^3 \Rightarrow m^3 - 19m + 30 = 0$

Then $m = -5, 2, 3$

But if $m = 2$ or 3 then the point, where the normal meets the curve will be $(am^2, -2am)$ where the curve does not exist. Therefore $m = -5$.

$\therefore \quad m + 6 = 1$

71 (1): $\left|\frac{1+x^2}{2x}\right| \leq 1 \Rightarrow |x| = 1 \Rightarrow x = \pm 1$

But $x = -1$ will not satisfy the equation.

72. (2): $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ and $\vec{a}\cdot\vec{b} = 0$ and $\vec{a}\cdot\vec{c} = 0$

$\vec{b}\cdot\vec{c} = |\vec{b}||\vec{c}|\cos\frac{\pi}{3} = \frac{1}{2}$.

$\therefore \quad |\vec{a} + \vec{b} + \vec{c}|^2 = 3 + 2\cdot 0 + 2\cdot 0 + 1 = 4$

$\therefore \quad |\vec{a} + \vec{b} + \vec{c}| = 2$

73. (8): L.H.L. $= \lim_{x \to \frac{\pi^-}{2}} f(x) = \frac{1}{2}$

and R.H.L. $= \lim_{x \to \frac{\pi^+}{2}} f(x) = \frac{b}{8}$

$\therefore \quad \frac{1}{2} = a = \frac{b}{8}$

74. (2):

$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\,dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\pi - x)\,dx + \int_{\frac{3\pi}{2}}^{2\pi} (x - 2\pi)\,dx = \frac{-\pi^2}{8}$

Math Archives 10 Best Problems

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE (Main & Advanced) Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE (Main & Advanced). In every issue of MT, challenging problems are offered with detailed solution. The readers' comments and suggestions regarding the problems and solutions offered are always welcome.

1. If $f(x)=\begin{cases}|x|, & \text{when } x\le 2\\ [x], & \text{when } x>2\end{cases}$, then

(a) $\lim\limits_{x\to 2^-} f(x)=-2$ (b) $\lim\limits_{x\to 2^+} f(x)=-2$

(c) $\lim\limits_{x\to 2^+} f(x)=f(2)$ (d) $\lim\limits_{x\to 2} f(x)$ does not exist

2. If $n\in N$, then $\lim\limits_{x\to\infty}\dfrac{x^n}{e^x}=0$

(a) when n is even only
(b) for no value of n
(c) for all values of n
(d) when n is odd only

3. If $y=\sin^{-1}\left[\dfrac{1-x^2}{1+x^2}\right]$, then $\dfrac{dy}{dx}=$

(a) $\dfrac{-2}{1+x^2}$ (b) $\dfrac{2}{1+x^2}$

(c) $\dfrac{1}{2+x^2}$ (d) $\dfrac{2}{2-x^2}$

4. The derivative of $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\dfrac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=0$ is

(a) $\dfrac{1}{8}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{1}{2}$ (d) 1

5. If $y=\tan^{-1}\left(\dfrac{\log(e/x^2)}{\log(ex^2)}\right)+\tan^{-1}\left(\dfrac{3+2\log x}{1-6\log x}\right)$, then $\dfrac{d^2y}{dx^2}$ is

(a) 2 (b) 1 (c) 0 (d) –1

6. Let $f(x)=\begin{cases}xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & ;x\neq 0.\\ 0 & ;x=0\end{cases}$

Test whether $f(x)$ is differentiable at $x=0$. Is it continuous at $x=0$? Justify.

7. A triangle ABC, right angled at C, with $CA = b$ and $CB = a$, moves such that the angular points A and B slide along x-axis and y-axis respectively. Find locus of C.

8. Prove that $\sin\theta\cdot\sec 3\theta=\dfrac{1}{2}(\tan 3\theta-\tan\theta)$ and hence find the sum to 'n' terms of the series $\sin\theta\cdot\sec 3\theta + \sin 3\theta\cdot\sec 3^2\theta + \sin 3^2\theta\cdot\sec 3^3\theta + ...$

9. Consider a real valued function $f(x)$ satisfying $2f(xy) = (f(x))^y + (f(y))^x \ \forall\ x, y \in R$ and $f(1) = p$ where $p \neq 1$, then find $(p-1)\sum\limits_{r=1}^{n} f(r)$.

10. A line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube. Prove that $\sum\limits_{r\in\{\alpha,\beta,\gamma,\delta\}}\cos^2(r)=\dfrac{4}{3}$

SOLUTIONS

1. (c) : $\lim\limits_{x\to 2^+} f(x)=\lim\limits_{h\to 0^+}[2+h]=2=f(2)$

$\lim\limits_{x\to 2^-} f(x)=\lim\limits_{h\to 0^-}|2-h|=2$

Hence limit exists.

2. (c) : Use L' Hospital rule

$\lim\limits_{x\to\infty}\dfrac{x^n}{e^x}=\dfrac{n!}{e^\infty}=0$

3. (a) : Put $x = \tan\theta \Rightarrow y=\dfrac{\pi}{2}-2\theta$

4. (b) : Let $P=\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta \Rightarrow P=\dfrac{\theta}{2}$

Hence, $\dfrac{dP}{dx}=\dfrac{1}{2(1+x^2)}$ ...(1)

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Let $Q=\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$, Put $x=\sin\phi$

$\Rightarrow \frac{dQ}{dx}=\frac{2}{\sqrt{1-x^2}}$ ...(2)

From (1) & (2)

$$\frac{dP}{dQ}=\frac{\left(\frac{1}{2(1+x^2)}\right)}{\left(\frac{2}{\sqrt{1-x^2}}\right)} \Rightarrow \left.\frac{dP}{dQ}\right|_{x=0}=\frac{1}{4}$$

5. **(c) :** In given function, put $\log x^2=\tan\theta$

$\therefore \quad y=\frac{\pi}{4}+\tan^{-1}3$

Hence, $\frac{d^2y}{dx^2}=0$

6. $f(x)=\begin{cases} xe^{\frac{-2}{x}} & ;x>0 \\ 0 & ;x=0 \\ x & x<0 \end{cases}$

$\therefore \quad f(0)=0$

$$\text{L.H.D}=f'\left(0^-\right)=\lim_{h\to0^-}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to0^-}\frac{-h}{-h}=1$$

$$\text{R.H.D}=f'(0^+)=\lim_{h\to0^+}\frac{f(0+h)-f(0)}{h}=\lim_{h\to0^+}\frac{he^{\frac{-2}{h}}}{h}$$

$=e^{-\infty}=0$

$\because$ L.H.D $\neq$ R.H.D

$\therefore$ f is not differentiable at $x=0$.

$\because$ L.H.D and R.H.D exist

$\therefore$ f is continuous at $x=0$.

7. Let $A=(p,0)$, $B=(0,q)$

Let $C(h,k)$ be the any point on the locus

$CB=a=\sqrt{h^2+(k-q)^2}$ ...(i)

$CA=b=\sqrt{(h-p)^2+k^2}$ ...(ii)

$AB=\sqrt{p^2+q^2}$ ...(iii)

$\because \quad \angle C=90^o$

$\therefore \quad AB^2=AC^2+BC^2$

$\Rightarrow p^2+q^2=a^2+b^2$ ...(iv)

From (i) and (ii)

$q=k\pm\sqrt{a^2-h^2}$, $p=h\pm\sqrt{b^2-k^2}$

$\therefore$ From (iv), Locus is $bx\pm ay=0$

8. $\frac{\sin\theta}{\cos3\theta}=\frac{\cos\theta\cdot\sin\theta}{\cos\theta\cdot\cos3\theta}$

$$=\frac{1}{2}\frac{\sin(3\theta-\theta)}{\cos\theta\cdot\cos3\theta}=\frac{1}{2}(\tan3\theta-\tan\theta)$$

$$\therefore \sum_{r=1}^{n}\sin3^{r-1}\theta\cdot\sec3^r\theta=\sum_{r=1}^{n}\frac{\sin3^{r-1}\theta.}{\cos3^r\theta}$$

$$=\frac{1}{2}\sum_{r=1}^{n}\frac{\sin(3^r\theta-3^{r-1}\theta)}{\cos3^r\theta\cdot\cos3^{r-1}\theta}=\frac{1}{2}\sum_{r=1}^{n}(\tan3^r\theta-\tan3^{r-1}\theta)$$

$$=\frac{1}{2}\left[\tan3^n\theta-\tan\theta\right]$$

9. $2f(xy)=(f(x))^y+(f(y))^x \ \forall\ x,y\in R$

Put $y=1$

$\Rightarrow \quad 2f(x)=f(x)+(f(1))^x$

$\Rightarrow \quad f(x)=p^x \quad (\because \ f(1)=p)$

$$\therefore \sum_{r=1}^{n}f(r)=\sum_{r=1}^{n}p^r=\frac{p^{n+1}-p}{p-1}$$

$$\Rightarrow (p-1)\sum_{r=1}^{n}f(r)=p^{n+1}-p$$

10. D.C's of $\overrightarrow{OP}=\left(\frac{1}{\sqrt3},\frac{1}{\sqrt3},\frac{1}{\sqrt3}\right)$

D.C's of $\overrightarrow{AR}=\left(-\frac{1}{\sqrt3},\frac{1}{\sqrt3},\frac{1}{\sqrt3}\right)$

D.C's of $\overrightarrow{BS}=\left(\frac{1}{\sqrt3},\frac{-1}{\sqrt3},\frac{1}{\sqrt3}\right)$

D.C's of $\overrightarrow{CQ}=\left(\frac{1}{\sqrt3},\frac{1}{\sqrt3},-\frac{1}{\sqrt3}\right)$

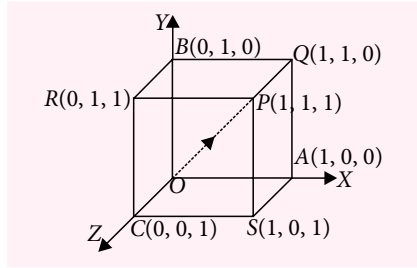
Let l, m, n be the D.C.'s of required line

$\therefore \quad \cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta$

$$=\frac{1}{3}((l+m+n)^2+(l-m+n)^2$$

$$+(l+m-n)^2+(-l+m+n)^2)$$

$=\frac{4}{3}$



JEE

PRACTICE PROBLEMS

Exam on 24th May

Advanced

1. Arithmetic mean of roots of a quadratic is 3.5 and their geometric mean is 2.5. The required quadratic equation is
(a) $4x^2 - 28x + 25 = 0$ (b) $4x^2 + 28x - 25 = 0$
(c) $x^2 - 14x + 25 = 0$ (d) $x^2 - 14x - 25 = 0$

2. For a complex number z, minimum value of $|z| + |z - 3|$ is
(a) 1/3 (b) 3
(c) 9 (d) none of these

3. Four couples (husband and wife) decide to form a committee of four members. Number of different committees that can be formed in which no couple is included, is
(a) 12 (b) 14
(c) 16 (d) 18

4. If $ax^2 + bx + 8 = 0$ has no distinct real roots then least value of $(4a + b)$ is (where a and b are real)
(a) -3 (b) -2
(c) -4 (d) none of these

5. The coefficient of x^{20} in expansion of
$(1+x^2)^{40}\left(x^2+2+\frac{1}{x^2}\right)^{-5}$ is
(a) ${}^{30}C_{10}$ (b) ${}^{30}C_{25}$
(c) 1 (d) none of these

6. $\sec^2\theta = \frac{4xy}{(x+y)^2}$, where $x, y \in R$ is true if and only if
(a) $x + y \neq 0$ (b) $x = y \neq 0$
(c) $y \neq 0$ (d) $x \neq y, y \neq 0$

7. If $\tan\frac{\pi}{9}$, x and $\tan\frac{5\pi}{18}$ are in A.P. and $\tan\frac{\pi}{9}$, y and $\tan\frac{7\pi}{18}$ are also in A.P., then
(a) $2x = y$ (b) $2y = x$
(c) $x = y$ (d) $y/x = 4$

8. General values of x for which $\sec x - 1 = (\sqrt{2} - 1)\tan x$ are
(a) $n\pi + \frac{\pi}{8}$ (b) $2n\pi, 2n\pi + \frac{\pi}{4}$
(c) $2n\pi$ (d) none of these

9. If $\log_{16}x^2 - \log_{16}x + \log_{16}k = 0$ has only one solution, then number of possible values of k is
(a) 2 (b) 3
(c) 1 (d) none of these

10. If A is the area and $2S$ is sum of three sides of a triangle; then
(a) $A \le \frac{S^2}{3\sqrt{3}}$ (b) $A \le \frac{S^2}{2}$
(c) $A > \frac{S^2}{\sqrt{3}}$ (d) none of these

11. If $\text{Re}\left(\frac{z-8}{z+6}\right) = 0$, then z lies on the curve
(a) $x^2 + y^2 + 6x - 8y = 0$
(b) $4x - 3y + 24 = 0$
(c) $x^2 + y^2 - 8 = 0$
(d) none of these

12. The solution set of the inequality
$\log_{\sin(\pi/3)} x^2 - 3x + 2 \ge 2$ is
(a) $\left(\frac{1}{2}, 2\right)$ (b) $\left(1, \frac{5}{2}\right)$
(c) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ (d) none of these

13. Set of values of x for which, $\sin x \cos^3 x > \cos x \sin^3 x$, $0 \le x \le \pi$ is
(a) $(0, \pi)$ (b) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$
(c) $\left(\frac{\pi}{4}, \pi\right)$ (d) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

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14. P is a point. Two tangents are drawn from it to the parabola $y^2 = 4x$ such that the slope of one tangent is three times the slope of the other. The locus of P is

(a) a straight line (b) a circle
(c) a parabola (d) an ellipse

15. Equation of the director circle of the ellipse $x^2 + 2y^2 + 2x - 12y + 15 = 0$ is

(a) $x^2 + y^2 + 2x - 6y + 4 = 0$
(b) $x^2 + y^2 + 2x - 12y + 4 = 0$
(c) $x^2 + y^2 + 2x - 6y - 4 = 0$
(d) $x^2 + y^2 + 2x - 12y - 4 = 0$

16. The sum of the digits at the ten's place of all the numbers formed with the help of 3, 4, 5, 6 taken all at a time is

(a) 432 (b) 108 (c) 36 (d) 18

17. If A (6, 3, 2), B (5, 1, 4), C (3, 4, 7), D (0, 2, 5) be four points, then projection of segment CD on the line AB is

(a) −13/3 (b) −13/7
(c) −3/13 (d) −7/13

18. The ratio in which the plane $\vec{r}\cdot(\hat{i}-2\hat{j}+3\hat{k})=17$ divides the line joining the points $-2\hat{i}+4\hat{j}+7\hat{k}$ and $3\hat{i}-5\hat{j}+8\hat{k}$ is

(a) 1 : 5 (b) 1 : 10 (c) 3 : 5 (d) 3 : 10

19. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}\cdot(\vec{b}+c)+\vec{b}\cdot(\vec{c}+\vec{a})+\vec{c}\cdot(\vec{a}+\vec{b})=0$ and $|\vec{a}|=1$, $|\vec{b}|=4, |\vec{c}|=8$, then $|\vec{a}+\vec{b}+\vec{c}|=$

(a) 13 (b) 81 (c) 9 (d) 5

20. If n is an odd natural number, then $\sum_{r=0}^{n} \frac{-1^r}{{}^nC_r}$ equals

(a) 0 (b) $1/n$
(c) $n/2^n$ (d) none of these

21. The determinant $\begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\cos\alpha & \sin\alpha & \cos\beta \end{vmatrix}$ is independent of

(a) α (b) β
(c) α and β (d) neither α nor β

22. $f(x)=\begin{cases} \dfrac{\sqrt{1+px}-\sqrt{1-px}}{x}, & -1\le x<0 \\ \dfrac{2x+1}{x-2}, & 0\le x\le 1 \end{cases}$

is continuous on [−1, 1], then p is

(a) −1 (b) −1/2
(c) 1/2 (d) 1

23. $\frac{d}{dx}\left(\sin^2\left(\cot^{-1}\frac{1}{\sqrt{\frac{1+x}{1-x}}}\right)\right)=$

(a) 0 (b) −1/2 (c) 1/2 (d) −1

24. The number of values of 'k' for which the equation $x^3 - 3x + k = 0$ has two different roots lying in the interval (0, 1) are

(a) 3 (b) 2
(c) infinitely many
(d) no value of k satisfies the requirement

25. If $\int_0^x f(t)dt = x + \int_x^1 tf(t)dt$, then the value of $f(1)$ is

(a) 1/2 (b) 0 (c) 1 (d) −1/2

26. If X follows a binomial distribution with parameters n and p, $0 < p < 1$. If $P(X = r)/P(X = n - r)$ is independent of n and r, then value of p is

(a) 1/2 (b) 1/3
(c) 1/4 (d) none of these

27. If in a ΔABC, $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then $\cos A : \cos B : \cos C =$

(a) 7 : 9 : 15 (b) 7 : 19 : 25
(c) 5 : 7 : 9 (d) 10 : 13 : 19

28. The equation of the plane through (2,3,–4) and (1, –1, 3) and parallel to x-axis is

(a) $7y - 4z - 5 = 0$ (b) $4y - 7z - 5 = 0$
(c) $4y + 7z + 5 = 0$ (d) $7y + 4z - 5 = 0$

29. $p \Rightarrow q$ can also be written as

(a) $p \Rightarrow \sim q$ (b) $\sim p \vee q$
(c) $p \wedge q$ (d) none of these

30. Median of 16, 10, 14, 11, 9, 8, 12, 6, 5 is

(a) 10 (b) 12 (c) 11 (d) 14

ANSWER KEYS

1. (a)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (b)
6. (b)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (a)
11. (d)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (a)
16. (b)	**17.** (a)	**18.** (d)	**19.** (c)	**20.** (a)
21. (a)	**22.** (b)	**23.** (c)	**24.** (d)	**25.** (a)
26. (a)	**27.** (b)	**28.** (d)	**29.** (b)	**30.** (a)

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10 BEST PROBLEMS *for* JEE Advanced

1. Given that

$$iz^2 = 1 + \frac{2}{z} + \frac{3}{z^2} + \frac{4}{z^3} + \frac{5}{z^4} + \ldots\ldots \text{ and } z = n \pm \sqrt{-i}.$$

Find the value of n.

2. Find the value of x by evaluating the given series

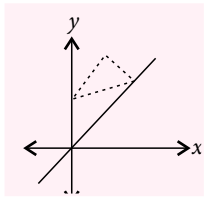
$$1 + \frac{1}{5} + \frac{1\times 3}{5\times 10} + \frac{1\times 3\times 5}{5\times 10\times 15} + \ldots\ldots\ldots\infty = \sqrt{x},\ x \in Q$$

3. If r_1, r_2, r_3, r_4, r_5 are the complex roots of the equation $x^5 - 3x^4 - 1 = 0$. Find the value of

$$\frac{1}{r_1^9} + \frac{1}{r_2^9} + \frac{1}{r_3^9} + \frac{1}{r_4^9} + \frac{1}{r_5^9}$$

4. Find the sum of all (distinct) complex values of c for which the polynomial $f_c(x) = x^4 - (c^2 - 7c + 11)x^2 + (18 - 21c + 8c^2 - c^3)$ has strictly less than four distinct complex zeroes.

5. The minimum possible perimeter of a triangle with one vertex at (3, 9), one anywhere on the y-axis and one anywhere on the line $y = x$ is



6. $\lim\limits_{n\to\infty} \sum\limits_{r=3}^{n} \frac{r^3 - 8}{r^3 + 8} =$

7. Inside a square $ABCD$, points P and Q are positioned so that $DP \parallel QB$ and $DP = PQ = QB$. Of all configurations that satisfy these requirements, what is the minimum possible value of $\angle ADP$, (in degrees)?

8. Find the value of $\int_0^{2n\pi} \max\left\{\sin x, \sin^{-1}(\sin x)\right\} dx$. Draw the graph of the same.

9. Let P be a point (other than the origin) lying on the parabola $y = x^2$. The normal line to the parabola at P will intersect the parabola at another point Q. The minimum possible value for the area bounded by the line PQ and the parabola is

10. Find the value of $\lim\limits_{x\to 0}\left(\int_0^1 (by + a(1-y))^x dy\right)^{\frac{1}{x}}$ where $b > a$.

SOLUTIONS

1. Multiplying both sides of the equation by z, we get

$$iz^3 = z + 2 + \frac{3}{z} + \frac{4}{z^2} + \ldots\ldots\ldots \quad \ldots(i)$$

and subtracting the original equation from (i) we get

$$iz^2(z-1) = z + 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \ldots\ldots\ldots$$

Using the formula for an infinite geometric series, we find

$$iz^2(z-1) = \frac{z}{1 - \frac{1}{z}} = \frac{z^2}{z-1}$$

Rearranging, we get

$$iz^2(z-1)^2 = z^2 \Rightarrow (z-1)^2 = \frac{1}{i} \Rightarrow z = 1 \pm \sqrt{-i}.$$

Thus $n = 1$

2. Each term is of the following form :

$$\frac{\prod\limits_{k=1}^{n}(2k-1)}{\prod\limits_{k=1}^{n}(5k)}$$

$$= \frac{\prod\limits_{k=1}^{n}(2k-1)\prod\limits_{k=1}^{n}(2k)}{(n!)5^n\prod\limits_{k=1}^{n}(2k)} = \frac{(2n)!}{(n!)5^n(n!)2^n} = \frac{1}{10^n}\binom{2n}{n}$$

Here $\binom{n}{r} = {}^nC_r$

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$$s=\sum_{n=0}^{\infty}\frac{1}{10^n}\binom{2n}{n}$$

Now, since the result is the square root of a rational number, let's find s^2. Using the Cauchy Product (with $\frac{1}{10}$ as the independent variable), we get the following formula.

$$s^2=\sum_{n=0}^{\infty}\frac{1}{10^n}\sum_{k=0}^{n}\binom{2k}{k}\binom{2(n-k)}{n-k}$$

Now it can be shown that for all whole numbers n, we have

$$\sum_{k=0}^{n}\binom{2k}{k}\binom{2(n-k)}{n-k}=4^n$$

Hence, we have

$$s^2=\sum_{n=0}^{\infty}\frac{1}{10^n}\times 4^n=\sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n=\frac{1}{1-\frac{2}{5}}=\frac{5}{3}$$

Thus, $x=\frac{5}{3}$

3. From the given polynomial, we have

$$\sum r_i=3 \text{ and } \sum r_i^{-1}=0$$

The sum of the reciprocals of the roots come from the fact that the polynomial with reciprocal roots has its coefficients reversed.

$x^5 - 3x^4 - 1 = 0 \Rightarrow x^4(x-3)=1$

$$\Rightarrow x^4=(x-3)^{-1} \Rightarrow x^{-9}=\frac{(x-3)^2}{x}$$

$$\sum r_i^{-9}=\sum\frac{(r_i-3)^2}{r_i}=\sum\frac{r_i^2-6r_i+9}{r_i}$$

$$=\sum(r_i-6+9r_i^{-1})=\sum r_i-\sum 6+9\sum r_i^{-1}$$

$= 3 - 5(6) + 0 = -27$

4. The polynomial $f_c(x)$ will fail to have four distinct complex zeros when the quadratic polynomial $g_c(x) = x^2 - (c^2 - 7c + 11)x + (18 - 21c + 8c^2 - c^3)$ either has repeated roots or has 0 as a root.

Case 1: One of the roots of $g_c(x)$ is zero precisely when

$c^3 - 8c^2 + 21c - 18 = (c-2)(c-3)^2 = 0$

and so precisely when $c = 2, 3$

Case 2 : The roots of $g_c(x)$ are repeated when its discriminant is zero, so that

$(c^2 - 7c + 11)^2 - 4(18 - 21c + 8c^2 - c^3) = 0$

$\Rightarrow c^4 - 10c^3 + 39c^2 - 70c + 49 = 0$

$\Rightarrow (c^2 - 5c + 7)^2 = 0$

and this happens for two distinct values of c which sum to 5.

Thus the sum of the possible values of c is

$2 + 3 + 5 = 10$.

5. Let A be the vertex (3,9), B be the vertex on the y-axis and C be the vertex on the line $y = x$. Also, let D (–3, 9) be the reflection of A in the y-axis and E(9, 3) be the reflection of A in the line $y = x$. Then $AB = BD$ and $AC = CE$, and thus the perimeter of ΔABC is equal to $DB + BC + CE$.

But the shortest distance between two points is a straight line, so

$DB + BC + CE \geq DE$

$$=\sqrt{(9-(-3))^2+(3-9)^2}=\sqrt{180}=6\sqrt{5}$$

This minimum can be obtained by then choosing B and C as the points of intersection of the line DE with the y-axis and the line $y = x$, respectively. This gives us the points $B\left(0,\frac{15}{2}\right)$ and $C(5, 5)$. This will yield a perimeter for ΔABC of $6\sqrt{5}$.

6.

$$\lim_{n\to\infty}\left(\frac{3^3-8}{3^3+8}\right)\left(\frac{4^3-8}{4^3+8}\right).....\left(\frac{n^3-8}{n^3+8}\right)$$

$$=\lim_{n\to\infty}\left(\frac{3-2}{3+2}.\frac{3^2+4+2(3)}{3^2+4-2(3)}\right)\left(\frac{4-2}{4+2}.\frac{4^2+4+2(4)}{4^2+4-2(4)}\right)$$

$$.....\left(\frac{n-2}{n+2}.\frac{n^2+4+2(n)}{n^2+4-2(n)}\right)$$

$$=\lim_{n\to\infty}\left\{\frac{3-2}{3+2}.\frac{4-2}{4+2}.\frac{5-2}{5+2}.....\frac{n-2}{n+2}\right\}$$

$$\left\{\frac{3^2+4+2(3)}{3^2+4-2(3)}.\frac{4^2+4+2(4)}{4^2+4-2(4)}.....\frac{n^2+4+2(n)}{n^2+4-2(n)}\right\}$$

$$=\lim_{n\to\infty}\left\{\frac{1\cdot2\cdot3\cdot4\cdot5\cdot6.....}{5\cdot6\cdot7\cdot8.....}\right\}\left\{\frac{19\cdot28\cdot39\cdot52\cdot63.....}{7\cdot12\cdot19\cdot28\cdot39\cdot52\cdot63.....}\right\}$$

$$=\lim_{n\to\infty}(1\cdot2\cdot3\cdot4)\left(\frac{1}{7\cdot12}\right)=\frac{2}{7}$$

7. Without loss of generality,

let the corners of the square be $A(0,2)$, $B(2,2)$, $C(2,0)$ and $D(0,0)$. Now let point P has coordinates (a, b); then by symmetry the coordinates of point Q must be $(2 - a, 2 - b)$. Then since $DP = PQ$, we have that

$a^2 + b^2 = (2 - 2a)^2 + (2 - 2b)^2$

$\Rightarrow 3a^2 + 3b^2 - 8a - 8b + 8 = 0$

$$\Rightarrow \left(a-\frac{4}{3}\right)^2+\left(b-\frac{4}{3}\right)^2=\frac{8}{9}$$

This means that P lies on a circle centered at $O\left(\frac{4}{3},\frac{4}{3}\right)$ with radius $r=\frac{2}{3}\sqrt{2}$.

Therefore, $\angle ADP$ will be minimized when DP is tangent to this circle.

Now by symmetry OD makes an angle of 45° with the x-axis and has length $\frac{4}{3}\sqrt{2}$.

Thus, $\angle PDO = 45° - \angle ADP$, and so

$$\sin(\angle PDO) = \sin(45° - \angle ADP) = \frac{OP}{OD} = \frac{\frac{2}{3}\sqrt{2}}{\frac{4}{3}\sqrt{2}} = \frac{1}{2}$$

This implies that $45° - \angle ADP = \sin^{-1}\left(\frac{1}{2}\right) = 30°$, and

So the minimum value for $\angle ADP$ is $45° - 30° = 15°$

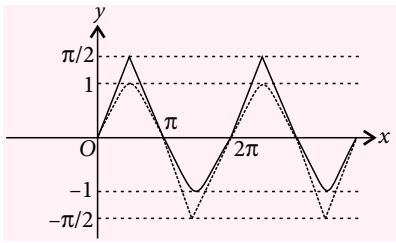
8. The integral can be divided as

$$= n\left[\int_0^{\frac{\pi}{2}} x\,dx + \int_{\frac{\pi}{2}}^{\pi} (\pi - x)\,dx + \int_{\pi}^{2\pi} \sin x\,dx\right]$$

$$= n\left[\frac{\pi^2}{8} + \frac{\pi^2}{2} - \frac{1}{2} \times \left(\pi^2 - \frac{\pi^2}{4}\right) - 2\right]$$

Solving, we get

$$I = n \times \frac{\pi^2 - 8}{4}$$



9. We take a point $P = (x_0, x_0^2)$ on the parabola. Then slope of tangent is $= 2x_0$

Hence slope of normal is $\frac{-1}{2x_0}$

So equation of normal is : $(x - x_0) = -2x_0(y - x_0^2)$

Solving it with the parabola, we get

$$x = x_0, y = \left(-\frac{1}{2x_0} - x_0\right)$$

So the other point is $Q = \left(-\frac{1}{2x_0} - x_0, \left(-\frac{1}{2x_0} - x_0\right)^2\right)$

Hence, the area bounded by PQ and the parabola is

$$\int_{\left(-\frac{1}{2x_0} - x_0\right)}^{x_0} \left(x_0^2 + \frac{1}{2} - \frac{x}{2x_0} - x^2\right) dx$$

(using the equation of normal)

Evaluating it, we get : $A = \frac{4}{3}\left(x_0 + \frac{1}{4x_0}\right)^3$

By applying AM-GM inequality, we get

$$\frac{x_0 + \frac{1}{4x_0}}{2} \geq \sqrt{\frac{1}{4}}$$

or $\left(x_0 + \frac{1}{4x_0}\right)^3 \geq 1$. Finally $A_{\min} = \frac{4}{3}$

10. Let $L = \lim_{x \to 0}\left(\int_0^1 (by + a(1-y))^x dy\right)^{\frac{1}{x}}$

Let $I = \int_0^1 \left(by + a(1-y)\right)^x dy$

Let $by + a(1-y) = t$, $(b - a)\,dy = dt$

$$\Rightarrow I = \int \frac{t^x}{(b-a)} dt = \frac{t^{x+1}}{(x+1)(b-a)}$$

$$I = \left[\frac{(by + a(1-y))^{x+1}}{(x+1)(b-a)}\right] \text{ [computed at } y = 1 \text{ and } y = 0]$$

$$I = \frac{b^{x+1} - a^{x+1}}{(x+1)(b-a)}$$

$$\Rightarrow L = \lim_{x \to 0}\left(\frac{b^{x+1} - a^{x+1}}{(x+1)(b-a)}\right)^{\frac{1}{x}}$$

$$ln(L) = \lim_{x \to 0} \frac{ln\left(\frac{\left(b^{x+1} - a^{x+1}\right)}{(x+1)(b-a)}\right)}{x}$$

As x approaches 0, denominator and numerator approaches 0.

Hence, we can use L-hospital's rule.

$$ln(L) = \lim_{x \to 0}\left(\frac{b^{x+1} ln(b) - a^{x+1} ln(a)}{(x+1)(b-a)} - \frac{1}{x+1}\right)$$

$$ln(L) = \frac{1}{b-a} ln\frac{b^b}{a^a} - 1$$

$$\Rightarrow L = \frac{1}{e}\left(\frac{b^b}{a^a}\right)^{\frac{1}{(b-a)}}$$

Practice Paper 2015

JEE Main

Exam on 4th April

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1. The number of common divisors of 10800 and 9000 is

(a) 49 (b) 36 (c) 25 (d) 64

2. A, B are two students in a group of n students. If the number of ways of assigning the n students to a line of n single rooms such that A and B are not in adjacent rooms is 3600, then n =

(a) 10 (b) 8 (c) 7 (d) 9

3. The sides AB, BC, CA of a triangle ABC have n, $n+1$, $n+2$ ($n \in N$, $n \geq 3$) interior points respectively on them. If the number of triangles formed by any three of these $(3n+3)$ points is 205, then n =

(a) 7 (b) 6 (c) 4 (d) 3

4. If the tangent and normal of a rectangular hyperbola $xy = 4$ cut off intercepts a_1, a_2 on x-axis and b_1, b_2 on y-axis respectively, then $a_1a_2 + b_1b_2$ =

(a) c^2 (b) $2c^2$ (c) 0 (d) $4c^2$

5. The length of common tangent of hyperbolas $\frac{x^2}{4}-\frac{y^2}{3}=1$ and $\frac{x^2}{3}-\frac{y^2}{4}+1=0$ is

(a) $\sqrt{\frac{2}{7}}$ (b) $7\sqrt{2}$ (c) 7 (d) $\frac{7}{\sqrt{2}}$

6. $\int_0^2 \left[\tan^{-1} x\right] dx + \int_0^2 \left[\cot^{-1} x\right] dx =$

(where [.] represents the greatest integer function)

(a) 1 – cot 2 (b) 1 + cot 2

(c) 2(1 + cot 2) (d) 2(1 – cot 2)

7. If $\int \frac{4e^x - 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B\ln(9e^{2x} - 4) + C$, then $A + B$ =

(a) $\frac{-9}{35}$ (b) $\frac{35}{36}$ (c) $\frac{9}{36}$ (d) None

8. If $\int f(x)\sin x \cos x \, dx = \frac{1}{2(b^2 - a^2)} \ln f(x) + c$, then $f(x)$ can be

(a) $\frac{1}{a^2\cos^2 x + b^2 \sin^2 x}$ (b) $\frac{1}{b^2\cos^2 x + a^2 \sin^2 x}$

(c) $a^2\cos^2 x + b^2 \sin^2 x$ (d) $b^2 \cos^2 x + a^2 \sin^2 x$

9. If $\sin^2 x - 2\sin x - 1 = 0$ has exactly four different solutions in $x \in [0, n\pi]$, then minimum value of 'n' can be ($n \in N$)

(a) 4 (b) 3 (c) 2 (d) 1

10. $P(a, b)$ is a point in first quadrant. If two circles which pass through 'P' and touch both the coordinate axes cut at right angles, then

(a) $a^2 - 6ab + b^2 = 0$ (b) $a^2 + 2ab - b^2 = 0$

(c) $a^2 - 4ab + b^2 = 0$ (d) $a^2 - 8ab + b^2 = 0$

11. AB is diameter of a semi circle K, C is an arbitrary point on the semicircle (other than A or B) and 'S' is the centre of the circle inscribed in ΔABC, then measure of

(a) angle ASB changes as C moves on K

(b) angle ASB is the same for all positions of 'C' but it cannot be determined without knowing the radius.

(c) angle ASB = 135° for all positions of C

(d) angle ASB = 150° for all positions of C

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12. $\int \frac{x^7}{(1-x^2)^5}\,dx =$

(a) $\frac{x^8}{(1-x^2)^4}+C$ (b) $\frac{x^8}{4(1-x^2)^8}+C$

(c) $\frac{x^8}{8(1-x^2)^4}+C$ (d) $\frac{x^8}{4(1-x^2)^4}+C$

13. If $f(x)=\sqrt{1+x\sqrt{1+(x+1)\sqrt{1+(x+2)(x+4)}}}$ then $\int_0^{100} f(x)\,dx =$

(a) 5010 (b) 5050
(c) 5100 (d) 5000

14. Degree of differential equation of the curve $y=a\left(1-e^{-\frac{x}{a}}\right)$, where '$a$' being the parameter is

(a) 1 (b) 2
(c) 3 (d) not defined

15. If solution of $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=x$ is given by $y^2=\frac{x^3}{\lambda}+C_1x+C_2$, then $\lambda =$

(a) 2 (b) 3
(c) 5 (d) 4

16. If $x \in R$, then the maximum value of $y=2(a-x)(x+\sqrt{x^2+b^2})$ is

(a) $a^2 + b^2$ (b) $a^2 - b^2$
(c) $a^2 + 2b^2$ (d) none of these

17. The number of three digit numbers of the form xyz such that $x < y$ and $z \le y$ is

(a) 176 (b) 278
(c) 276 (d) 240

18. If $(1.5)^{30} = k$, then the value of $\sum_{n=2}^{29}(1.5)^n$, is

(a) $2k - 3$ (b) $k + 1$
(c) $2k + 7$ (d) $2k - 9/2$

19. The set of values of x for which $\frac{1}{1+\sqrt{x}}, \frac{1}{1-x}, \frac{1}{1-\sqrt{x}}$ are in A.P. is given by

(a) $(-\infty, \infty)$ (b) $(1, \infty)$
(c) $[0, \infty) - \{1\}$ (d) $[0, \infty)$

20. The ratio of n^{th} terms of two A.P.'s is $(14n - 6) : (8n + 23)$. Then the ratio of their sum of first m terms is

(a) $\frac{4m+4}{7m+24}$ (b) $\frac{7m-1}{4m+24}$

(c) $\frac{7m-1}{4m-24}$ (d) $\frac{7m+1}{4m+27}$

21. If $A=\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$, then $A^{16} =$

(a) $\begin{bmatrix}0 & 256\\ 256 & 0\end{bmatrix}$ (b) $\begin{bmatrix}256 & 0\\ 0 & 256\end{bmatrix}$

(c) $\begin{bmatrix}-16 & 0\\ 0 & -16\end{bmatrix}$ (d) $\begin{bmatrix}0 & 16\\ 16 & 0\end{bmatrix}$

22. Tangents are drawn to the circle $x^2 + y^2 = 12$ at the point where it is met by the circle $x^2 + y^2 - 5x + 3y - 2 = 0$. The point of intersection of these tangents is

(a) $\left(\frac{18}{5}, 6\right)$ (b) $\left(6, \frac{-18}{5}\right)$

(c) $(4, -5)$ (d) $(5, -3)$

23. Let a, b and c be positive real numbers. Then the following system of equations in x, y and z

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1, \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1, -\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

has

(a) no solution (b) unique solution
(c) infinitely many solutions
(d) finitely many solutions

24. If the line $lx + my + n = 0$ cuts the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in points whose eccentric angles differ by $\frac{\pi}{2}$, then $\frac{a^2l^2+b^2m^2}{n^2} =$

(a) 1 (b) 2 (c) 4 (d) 3/2

25. If $F(x)=\lim_{n\to\infty} n^2(x^{1/n}-x^{1/(n+1)}), x>0$ then $\int xF(x)\,dx$ is equal to

(a) $\frac{x^2}{2}+\ln x+c$ (b) $-\frac{x^2}{4}+\ln x+x+\frac{x^2}{2}+c$

(c) $\frac{x^3}{3}+x\ln x+c$ (d) $\frac{x^2}{2}.\ln x-\frac{x^2}{4}+c$

26. The number of zero's at the end of 60! is

(a) 14 (b) 15 (c) 16 (d) 10

27. The number formed by last two digits of the number $(17)^{256}$ is

(a) 81 (b) 82 (c) 91 (d) 93

28. An ellipse of eccentricity $\frac{2\sqrt{2}}{3}$ is inscribed in a circle and a point with in the circle is chosen at random. The probability that this point lies outside the ellipse, is

(a) 1/9 (b) 4/9 (c) 1/3 (d) 2/3

29. In the binomial distribution $\left(\frac{3}{5}+\frac{2}{5}\right)^5$ if variance is V and standard deviation S, then

(a) $S = V^2$ (b) $4(S - V^2) = V^2$

(c) $5(V^2 - S^2) = S^2$ (d) none of these

30. If C_r stands for ${}^nC_r$, then the sum of the series (where n is an even positive integer)

$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}[C_0^2 - 2C_1^2 + 3C_2^2 + \ldots.. + (-1)^n(n+1)C_n^2]$$

is equal to

(a) 0 (b) $(-1)^n/2(n + 1)$

(c) $(-1)^n(n + 2)$ (d) None of these

31. The coefficient of x^{3l+2}, m the expansion of $(a + x)^l (b + x)^{l+1} (c + x)^{l+2}$ (l is a positive integer) is

(a) $l(a + b + c)$ (b) $l(a + b + c) + b + 2c$

(c) $l(a + b + c) + a + 2b + 3c$

(d) None of above

32. If n is a natural number , then $n^n - {}^nC_1(n - 1)^n + {}^nC_2(n - 2)^n - {}^nC_3(n - 3)^n + \ldots..$ must be equal to

(a) $n!$ (b) $(n!)^n$ (c) $n^n!$ (d) 0

33. Three normals are drawn to the curve $y^2 = x$ from a point $(c, 0)$. Out of three, one is always on x-axis. If two other normals are perpendicular to each other, then the value of c is

(a) 3/4 (b) 1/2 (c) 3/2 (d) 2

34. If area of triangle formed by tangents from the point (x_1, y_1) to the parabola $y^2 = 4ax$ and their chord of contact is

(a) $\frac{(y_1^2 - 4ax_1)^{3/2}}{2a^2}$ (b) $\frac{(y_1^2 - 4ax_1)^{3/3}}{a^2}$

(c) $\frac{(y_1^2 - 4ax_1)^{3/2}}{2a}$ (d) none of these

35. A normal to the parabola $y^2 = 4ax$ with slope 'm' touches the rectangular hyperbola $x^2 - y^2 = a^2$ if,

(a) $m^6 + 4m^4 - 3m^2 + 1 = 0$

(b) $m^6 - 4m^4 + 3m^2 - 1 = 0$

(c) $m^6 + 4m^4 + 3m^2 + 1 = 0$

(d) $m^6 - 4m^4 - 3m^2 + 1 = 0$

36. The number of ways of selecting 10 balls out of an unlimited number of white, red, blue and green balls is

(a) 270 (b) 84 (c) 286 (d) 86

37. A five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways in which this can be done is

(a) 216 (b) 240 (c) 600 (d) 3125

38. The number of ways in which 7 persons can be seated at a round table if two particular persons are not to sit together is

(a) 120 (b) 480

(c) 600 (d) 720

39. A is a set containing 'n' elements. A subset P of A is chosen. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen. The number of ways of choosing P and Q, so that $P \cap Q$ contain exactly two elements is

(a) $9\ {}^nC_2$ (b) $3^n - {}^nC_2$

(c) $2\ {}^nC_n$ (d) ${}^nC_2 \times 3^{n-2}$

40. The number of +ve integer satisfying the inequality ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 100$ is

(a) nine (b) eight

(c) five (d) None of these

41. The solution of the differential equation

$\frac{dy}{dx} = y + \int_0^1 y\,dx$, given that $y = 1$, when $x = 0$ is

(a) $\frac{2e^x - e + 1}{3 - e}$ (b) $\frac{2e^x + e + 1}{3 + e}$

(c) $\frac{2e^x + e + 1}{3 - e}$ (d) $\frac{e^x - 2e + 1}{3 - e}$

42. The polynomial function $f(x)$ of degree 6, which satisfies $\lim_{x\to 0}\left(1 + \frac{f(x)}{x^3}\right)^{1/x} = e^2$ and has local maximum at $x = 1$ and local minimum at $x = 0$ and $x = 2$ is

(a) $f(x) = 2x^4 - 12x^5 + \frac{2}{3}x^6$

(b) $f(x) = 2x^4 - \frac{12}{5}x^5 + 2x^6$

(c) $f(x) = 2x^4 - \frac{12}{5}x^5 + \frac{2}{3}x^6$

(d) $f(x) = x^4 + \frac{12}{5}x^5 - \frac{2}{3}x^6$

43. If $f(x)=x+\int_0^1 (xy^2+x^2y)f(y)\,dy$, then $f(x)$ is

(a) $\frac{180}{119}x+\frac{80}{119}x^2$ (b) $\frac{180}{119}x-\frac{80}{119}x^2$

(c) $\frac{-180}{119}x+\frac{80}{119}x^2$ (d) $-\left(\frac{180}{119}x+\frac{80}{119}x^2\right)$

44. A periodic function with period '1' is integrable over any finite interval. Also for two real numbers a, b and for two unequal non-zero positive integers m & n $\int_a^{a+n} f(x)dx=\int_b^{b+m} f(x)dx$, then the value of $\int_m^n f(x)dx$ is

(a) 0 (b) 1/2 (c) 2 (d) mn

45. If $y=\text{arc}\cos\sqrt{\left(\frac{\cos 3x}{\cos^3 x}\right)}$, then $\frac{dy}{dx}$ is

(a) $\sqrt{\frac{3}{\cos 4x+\cos 2x}}$ (b) $\sqrt{\frac{2}{\cos 4x+\cos 2x}}$

(c) $\sqrt{\frac{6}{\cos 4x+\cos 2x}}$ (d) $\sqrt{\frac{6}{\cos 4x-\cos 2x}}$

SOLUTIONS

1. (b) : $9000 = 2^3 \times 5^3 \times 3^2$ and $10800 = 2^4 \times 3^3 \times 5^2$. A common divisor is of the form $2^{\alpha_1}3^{\alpha_2}5^{\alpha_3}$, where $0 \le \alpha_1 \le 3$, $0 \le \alpha_2 \le 2$, $0 \le \alpha_3 \le 2$. Hence, number of common divisors is $(3 + 1)(2 + 1)(2 + 1) = 36$.

2. (c) : Treating 2 adjacent rooms as a single unit, number of ways of assigning n rooms to n students is $2(n - 1)!$ and total number of ways of assigning rooms is $n!$. Hence $n! - 2(n - 1)! = 3600$ (given)
i.e., $(n - 2)(n - 1)! = 5(6!) \Rightarrow n = 7$

3. (d) : There are totally $(3n + 3)$ points and if none of them are collinear, we can form ${}^{(3n+3)}C_3$ triangles. But by joining any 3 on AB (or AC or BC), we do not get any triangle, hence given
${}^{(3n+3)}C_3 - ({}^nC_3 + {}^{(n+1)}C_3 + {}^{(n+2)}C_3) = 205 \Rightarrow n = 3$

4. (c) : Tangent is perpendicular to normal at any point and slope of a line $= -\frac{\text{intercept on } y\text{-axis}}{\text{intercept on } x\text{-axis}}$

5. (b) : $y = mx + \sqrt{4m^2 - 3}$ is a tangent to 1st hyperbola and it becomes a common tangent if $m = \pm 1$. Hence common tangents are $y = \pm x + 1$. The points of contact are $(-4, -3)$ and $(3, 4)$, hence length of common tangent is $7\sqrt{2}$

6. (c) : $[\tan^{-1}x]=\begin{cases}0; & 0\le x<\tan 1\\ 1; & x\ge\tan 1\end{cases}$

$[\cot^{-1}x]=\begin{cases}1; & 0\le x\le\cot 1\\ 0; & x>\cot 1\end{cases}$

$$\int_{\tan 1}^{2} 1\,dx+\int_0^{\cot 1} 1\,dx=2-\tan 1+\cot 1=2(1+\cot 2)$$

7. (b) : $4e^x-6e^{-x}=A(9e^x-4e^{-x})+B\frac{d}{dx}(9e^x-4e^{-x})$

8. (b) : $f(x)\sin x\cos x=\frac{1}{2(b^2-a^2)}\frac{f'(x)}{f(x)}$

$\Rightarrow 2b^2\sin x\cos x-2a^2\sin x\cos x=\frac{f'(x)}{(f(x))^2}$

Integrating both sides w.r.t 'x', we get

$$-b^2\cos^2 x-a^2\sin^2 x=\frac{-1}{f(x)}$$

$\Rightarrow f(x)=\frac{1}{a^2\sin^2 x+b^2\cos^2 x}$

9. (a) : $(\sin x-1)^2=2 \Rightarrow \sin x-1=\pm\sqrt{2}$

$\Rightarrow \sin x=1-\sqrt{2}$ (since sin$x$ is not greater than 1)
2 solutions in $[0, 2\pi]$ and two more in $[2\pi, 4\pi]$

10. (c) : Let the circle be $x^2+y^2-2\sqrt{c}x-2\sqrt{c}y+c=0$
Since it passes through (a, b), we have

$$c-2\sqrt{c}(a+b)+a^2+b^2=0$$

$\Rightarrow \sqrt{c_1}+\sqrt{c_2}=2(a+b);\ \sqrt{c_1}\sqrt{c_2}=a^2+b^2$

$\because$ The circles are orthogonal

$\therefore 4\sqrt{c_1}\sqrt{c_2}=c_1+c_2, \Rightarrow a^2 - 4ab + b^2 = 0$

11. (c) : $\angle C=90°,\ \angle ASB=90°+\frac{C}{2}=135°$

12. (c) : $\int\frac{\frac{1}{x^3}}{\left(\frac{1}{x^2}-1\right)^5}dx$, Let $\frac{1}{x^2}-1=t$

$$\Rightarrow -\frac{1}{2}\int t^{-5}dt=-\frac{1}{2}\left(\frac{t^{-4}}{-4}\right)=\frac{1}{8\left(\frac{1}{x^2}-1\right)^4}+C=\frac{x^8}{8(1-x^2)^4}+C$$

13. (c) : $f(x) = (x + 1),\ x \in [0, 100]$

14. (d) : *D.E.* formed after eliminating a is

$$y = -\frac{x}{\log\left(\frac{dy}{dx}\right)}\left(1-\frac{dy}{dx}\right)$$

Hence degree is not defined.

15. (b) : $\frac{d}{dx}\left(y\left(\frac{dy}{dx}\right)\right) = x$

$$\Rightarrow \quad y\frac{dy}{dx} = \frac{x^2}{2} + P \quad \therefore \quad \frac{y^2}{2} = \frac{1}{6}x^3 + Px + q$$

$$\therefore \quad y^2 = \frac{x^3}{3} + C_1 x + C_2 \ \therefore \ \lambda = 3$$

16. (a) : Let $x + \sqrt{x^2+b^2} = t$

$$\Rightarrow \quad \sqrt{x^2+b^2} - x = \frac{b^2}{t}$$

$$\Rightarrow \quad 2\sqrt{x^2+b^2} = t + \frac{b^2}{t}; \quad 2x = t - \frac{b^2}{t}.$$

$$\therefore \quad (2a-2x)(x+\sqrt{x^2+b^2})$$

$$= \left(2a - t + \frac{b^2}{t}\right)t = 2at - t^2 + b^2$$

$$= (a^2+b^2) - (t-a)^2 \le a^2 + b^2$$

17. (c) : If zero is included, it will be at z, then no. of three digit numbers $= {}^9C_2$. If zero is excluded,

then $\begin{cases} x, y, z \text{ all diff} \Rightarrow {}^9C_3 \times 2! \\ x = z < y \Rightarrow {}^9C_2 \\ x < y = z \Rightarrow {}^9C_2 \end{cases}$

$\therefore$ Total number of ways = 276

18. (d): $S = \sum_{r=0}^{29} (1.5)^r - 1 - 1.5 = \frac{(1.5)^{30}-1}{1.5-1} - 2.5 = 2k - \frac{9}{2}$

19. (c) : $\frac{1}{1+\sqrt{x}} + \frac{1}{1-\sqrt{x}} = \frac{2}{1-x}$ is true for all x and it is defined for $x \ge 0$, $x \ne 1$

20. (d) : $\frac{T_n}{T_n'} = \frac{a_1+(n-1)d_1}{a_2+(n-1)d_2} = \frac{2a_1+(2n-2)d_1}{2a_2+(2n-2)d_2} = \frac{14n-6}{8n+23}$

Put $2n - 2 = m - 1$ in both sides,

$$\Rightarrow \quad \frac{2a_1+(m-1)d_1}{2a_2+(m-1)d_2} = \frac{7m+1}{4m+27}$$

21. (b) : $A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$, $A^4 = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$, $A^8 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$;

$$A^{16} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$$

22. (b) : Common chord is $5x - 3y - 10 = 0$. The chord of contact of required point w.r.t circle $x^2 + y^2 = 12$ is also the common chord of given two circles. Chord of contact of $P(x_1, y_1)$ is $xx_1 + yy_1 - 12 = 0$

$$\Rightarrow \quad \frac{x_1}{5} = \frac{y_1}{-3} = \frac{-12}{-10}$$

23. (d) : Let $\frac{x^2}{a^2} = X$, $\frac{y^2}{b^2} = Y$, $\frac{z^2}{c^2} = Z \Rightarrow X + Y - Z = 1$, $X - Y + Z = 1$, $-X + Y + Z = 1$. On solving $X = Y = Z = 1$

$\Rightarrow$ $x = \pm a$, $y = \pm b$, $z = \pm c$ $\Rightarrow$ 8 solutions

24. (b) : Let the point of intersection of the line and the ellipse be $(a\cos\theta, b\sin\theta)$ and $\left(a\cos\left(\frac{\pi}{2}+\theta\right), b\sin\left(\frac{\pi}{2}+\theta\right)\right)$. Since they lie on the given line $lx + my + n = 0$,

$la\cos\theta + mb\sin\theta + n = 0 \Rightarrow la\cos\theta + mb\sin\theta = n$ and $-la\sin\theta + mb\cos\theta + n = 0 \Rightarrow la\sin\theta - mb\cos\theta = n$.

Squaring and adding, we get

$$a^2l^2 + b^2m^2 = 2n^2 \ \Rightarrow \ \frac{a^2l^2+b^2m^2}{n^2} = 2$$

25. (d) : $F(x) = \lim_{n\to\infty} n^2 x^{1/(n+1)}\left[x^{\frac{1}{n(n+1)}} - 1\right]$

$$= \lim_{n\to\infty} \frac{x^{1/(n+1)}\left[x^{1/n(n+1)} - 1\right]}{\frac{1}{n(n+1)} \times \frac{n(n+1)}{n^2}} = \log x$$

Hence,

$$\int xF(x)\,dx = \int x\log x\,dx = \frac{x^2}{2}\log x - \frac{1}{4}x^2 + c$$

26. (a) : Number of zero's at the end of 60! = exponent of 10 in 60!

$= \min\{E_2(60!), E_5(60!)\} = E_5(60!) = 14$

27. (a) : $(17)^{256} = (289)^{128} = (300 - 11)^{128}$

$= {}^{128}C_0(-11)^{128} + 100m$, for some integer m

$= 11^{128} + 100m = (10 + 1)^{128} + 100m$

$= {}^{128}C_0 1^{128} + {}^{128}C_1 10 + 100m_1 + 100m$ for some integer m_1

$= 1 + 1280 + 100k$, $\qquad (m + m_1 = k)$

$= 1281 + 100k$

Hence the required number is 81.

28. (d) : Let the radius of the circle be a, then the major axis of the inscribed ellipse is of length $2a$.

The required probability $= \dfrac{\pi a^2 - \pi a^2\sqrt{1-e^2}}{\pi a^2}$

$= 1-\sqrt{1-e^2} = 1-\sqrt{1-\dfrac{8}{9}} = \dfrac{2}{3}$

(area of ellipse $= \pi ab = \pi a \cdot a\sqrt{1-e^2}$,

'e' being eccentricity)

29. (c) : $V = npq = 5.\dfrac{3}{5}.\dfrac{2}{5} = \dfrac{6}{5}$

$S = \sqrt{\dfrac{6}{5}}$ and $5(V^2 - S^2) = 5\left(\dfrac{36}{25} - \dfrac{6}{5}\right)$

$= 5\left(\dfrac{36}{25} - \dfrac{30}{25}\right) = \dfrac{6}{5} = S^2$

30. (d) : Let us test the choice for $n = 2$

For $n = 2$, the series

$= \dfrac{2(1!)(1!)}{2!}[{}^2C_0^2 - 2.{}^2C_1{}^2 + 3.{}^2C_2^2] = -4$

Now observe the table

Choice	Value at $n = 2$
(a)	0
(b)	–3
(c)	4

Since none of the choices become – 4, the correct choice must be (d).

31. (b) : Given expression = $(x + a)\ (x + a) \ldots l$ times $(x + b)\ (x + b)$ .....$(l + 1)$ times $\times\ (x + c)\ (x + c) \ldots (l + 2)$ times

$= x^{3l+3} + [al + b(l+1) + c(l+2)]\ x^{3l+2} + \ldots$

$\Rightarrow$ co-efficient of x^{3l+2} is $(a + b + c)l + b + 2c$

Choice (b) is correct.

32. (a) : $n^n - {}^nC_1(n-1)^n + {}^nC_2(n-2)^n + \ldots.$

Number of ways of districting n objects in n district cells such that no cell remains empty = $n!$

33. (a) : Normal at $(at^2, 2at)$ is $y + tx = 2at + at^3 \left(a = \dfrac{1}{4}\right)$

If this passes through $(c, 0)$

We have, $ct = 2at + at^3 = \dfrac{t}{2} + \dfrac{t^3}{4}$

$\Rightarrow\ t = 0$ or $t^2 = 4c - 2$

If $t = 0$, the point at which the normal is drawn is $(0, 0)$. If $t \neq 0$, then the two values of t represents slope of normals through $(c, 0)$

If these normals are perpendicular, then

$(-t_1)(-t_2) = -1 \Rightarrow t_1t_2 = -1 \Rightarrow (\sqrt{4c-2})(-\sqrt{4c-2}) = -1$

$\Rightarrow\ c = \dfrac{3}{4}$

34. (c) : Let $A(x_1, y_1)$ be any point outside the parabola and $B(\alpha, \beta)$, $C(\alpha', \beta')$ be the points of contact of tangents from point A. Eq. of chord BC, $yy_1 = 2a(x + x_1)$

Lengths of $\perp$ AL from A to BC

$AL = \dfrac{2a(x_1 + x_1) - y_1y_1}{\sqrt{y_1^2 + 4a^2}} = \dfrac{y_1^2 - 4ax_1}{\sqrt{y_1^2 + 4a^2}}$

Area of $\Delta ABC = \dfrac{1}{2}\ AL \times BC$

We get, $\dfrac{(y_1^2 - 4ax_1)^{3/2}}{2a}$

35. (c) : Equation of the normal to the parabola $y^2 = 4ax$ with slope m is $y = mx - 2am - am^3$. It touches the rectangular hyperbola $x^2 - y^2 = a^2$ if

$(-2am - am^3)^2 = a^2(m^2 - 1)$

$\Rightarrow\ a^2m^2(2 + m^2)^2 = a^2(m^2 - 1)$

$\Rightarrow\ m^2(m^4 + 4m^2 + 4) = m^2 - 1$

$\Rightarrow\ m^6 + 4m^4 + 3m^2 + 1 = 0$

36. (c) : Let x_1, x_2, x_3 and x_4, be the no. of white, red, blue and green balls that are selected. Then $x_1 + x_2 + x_3 + x_4 = 10$. The required no. of ways

= coefficient of y^{10} in $(1 + y + y^2 + y^3 + \ldots.)^4$

= coefficient of y^{10} in $(1 - y)^{-4}$

= coefficient of y^{10} in $(1 + {}^4C_1\ y + {}^5C_2\ y^2 + {}^6C_3\ y^3 + \ldots + {}^{13}C_{10}\ y^{10}\ldots.)$

$= {}^{13}C_3 = \dfrac{13 \times 12 \times 11}{2 \times 3} = 286$

37. (a) : The sum of the numerals 0, 1, 2, 3, 4 and 5 is 15. We know that a five digit number is divisible by 3 if and only if the sum of its digits is divisible by 3. Therefore, we should not use either 0 or 3 while forming the five digit number. If we do not use 0, then the remaining digits can be arranged in ${}^5P_5 = 5! = 120$ ways. If we do not use 3, then the remaining digits can be arranged in ${}^5P_5 - {}^4P_4 = 5! - 4! = 120 - 24 = 96$ ways to obtain a five digit number. Thus, the total no. of such 5 digit numbers is 120 + 96 = 216.

38. (b) : 6! – 2! 5! = 480

39. (d) : $A = \{a_1, a_2, \ldots., a_n\}$

(i) $a_i \in P, a_i \in Q$

(ii) $a_i \in P, a_i \notin Q$

(iii) $a_i \notin P, a_i \in Q$

(iv) $a_i \notin P, a_i \notin Q$

$P \cap Q$ contains exactly two elements, taking 2 elements in (i) and $(n-2)$ elements in (ii) or (iii) or (iv)

No. of ways $= {}^{n}C_2 \times 3^{n-2}$

40. (b) : ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 100$

$$\Rightarrow {}^{n+1}C_3 - {}^{n+1}C_2 \le 100 \Rightarrow \frac{1}{6}(n+1)n(n-1) - \frac{n(n+1)}{2} \le 100$$

$$\Rightarrow \frac{1}{6}n(n+1)\{n-1-3\} \le 100 \Rightarrow n(n+1)(n-4) \le 600$$

It is true for $n = 2, 3, 4, 5, 6, 7, 8, 9$

41. (a) : Let $y = f(x) \Rightarrow \frac{f''(x)}{f'(x)} = 1$

$$\Rightarrow f(x) = (Ce^x + D)$$

At $x = 0$, $y = f(x) = 1 \Rightarrow C = \frac{2}{3-e}$, $D = 1 - \frac{2}{3-e}$

42. (c) : $f(x) = ax^4 + bx^5 + cx^6$

$a = 2$, $f'(x) = x^3(8 + 5bx + 6cx^2)$

$b = -12/5$, $c = 2/3$

43. (a) : $A = \int_0^1 y^2 f(y)dy$, $B = \int_0^1 y f(y)dy$

$f(x) = x + Ax + Bx^2$

$$\Rightarrow A = \frac{61}{119}, B = \frac{80}{119}$$

44. (a) : $f(1 + x) = f(x)$

$$\therefore \int_a^{a+n} f(x)dx = \int_0^n f(x)dx = n\int_0^1 f(x)dx$$

similarly only $m\int_0^1 f(x)dx$

$$\Rightarrow (n-m)\int_0^1 f(x)dx = 0 \Rightarrow \int_0^1 f(x)dx = 0$$

$$\int_m^n f(x)dx = \int_0^{n-m} f(m+x)dx = \int_0^{n-m} f(x)dx$$

$$= (n-m)\int_0^1 f(x)dx = 0$$

45. (c) : $\cos^2 y = \frac{\cos 3x}{\cos^3 x}$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{3}\sec^2 x}{\cos y}$$

❍❍

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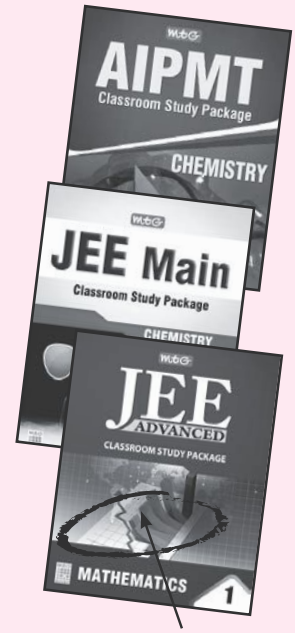

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Exam on : 18th, 19th April

WBJEE PRACTICE PAPER

CATEGORY-1

For each correct answer one mark will be awarded, whereas, for each wrong answer, 25% of total marks (1/4) will be deducted. If candidates marks more than one answer, negative marking will be done.

1. In an A.P., the first term is 2 and the sum of first five terms is 5. Then the 31st term is

(a) 13 (b) 17 (c) –13 (d) 27/2

2. If the second and fifth terms of a G.P. are 24 and 3 respectively, the sum of first six terms is

(a) 181 (b) $\frac{181}{2}$ (c) 189 (d) $\frac{189}{2}$

3. If two positive numbers are in ratio $3+2\sqrt{2}:3-2\sqrt{2}$ then the ratio between their A.M. and G.M is

(a) 6 : 1 (b) 3 : 2 (c) 2 : 1 (d) 3 : 1

4. Consider an infinite G.P. with first term a and common ratio r. If the sum is 4 and the second term is 3/4, then

(a) $a = 2, r = 3/8$ (b) $a = 4/7, r = 3/7$
(c) $a = 3/2, r = 1/2$ (d) $a = 3, r = 1/4$

5. If $\log_e 5, \log_e(5^x - 1)$ and $\log_e\left(5^x - \frac{11}{5}\right)$ are in A.P., then the values of x are

(a) $\log_5 4, \log_5 3$ (b) $\log_3 4, \log_4 3$
(c) $\log_3 4, \log_3 5$ (d) 12, 6

6. If $\log_2(9^{x-1} + 7) - \log_2(3^{x-1} + 1) = 2$, then values of x are

(a) 1, 2 (b) 0, 2
(c) 0, 1 (d) 1, 4

7. If the complex numbers z_1, z_2 and z_3 denote the vertices of an isosceles triangle, right angled at z_1, then $(z_1 - z_2)^2 + (z_1 - z_3)^2$ is equal to

(a) 0 (b) $(z_2 + z_3)^2$
(c) 2 (d) 3

8. Let x_1 and y_1 be real numbers. If z_1 and z_2 are complex numbers such that $|z_1| = |z_2| = 4$, then $|x_1z_1 - y_1z_2|^2 + |y_1z_1 + x_1z_2|^2 =$

(a) $32(x_1^2 + y_1^2)$ (b) $16(x_1^2 + y_1^2)$
(c) $4(x_1^2 + y_1^2)$ (d) none of these

9. If α and β are the roots of $x^2 - ax + b^2 = 0$, then $\alpha^2 + \beta^2$ is equal to

(a) $a^2 + 2b^2$ (b) $a^2 - 2b^2$
(c) $a^2 - 2b$ (d) $a^2 + 2b$

10. The equation whose roots are the squares of the roots of the equation $2x^2 + 3x + 1 = 0$ is

(a) $4x^2 + 5x + 1 = 0$ (b) $4x^2 - x + 1 = 0$
(c) $4x^2 - 5x - 1 = 0$ (d) $4x^2 - 5x + 1 = 0$

11. Let T_n denote the number of triangles which can be formed by using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 36$, then n is equal to

(a) 2 (b) 5 (c) 6 (d) 9

12. How many 5 digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once?

(a) 335 (b) 336 (c) 338 (d) 337

13. If n is a natural number, then

(a) $1^2 + 2^2 + \ldots + n^2 < \frac{n^3}{3}$

(b) $1^2 + 2^2 + \ldots + n^2 = \frac{n^3}{3}$

(c) $1^2 + 2^2 + \ldots + n^2 > n^3$

(d) $1^2 + 2^2 + \ldots + n^2 > \frac{n^3}{3}$

By : Sankar Ghosh, HOD(Math), Takshyashila. Mob : 09831244397.

14. The remainder obtained when $1! + 2! + 3! + ... + 11!$ is divided by 12, is

(a) 9 (b) 8 (c) 7 (d) 6

15. If 21^{st} and 22^{nd} terms in the expansion of $(1 + x)^{44}$ are equal then x is equal to

(a) $\frac{8}{7}$ (b) $\frac{21}{22}$ (c) $\frac{7}{8}$ (d) $\frac{23}{24}$

16. The middle term in the expansion of $\left(\frac{10}{x}+\frac{x}{10}\right)^{10}$ is

(a) ${}^{10}C_5$ (b) ${}^{10}C_6$

(c) ${}^{10}C_5 \frac{1}{x^{10}}$ (d) ${}^{10}C_5\, x^{10}$

17. The expression ${}^nC_0 + 2{}^nC_1 + 3{}^nC_2 + ... + (n + 1){}^nC_n$ is equal to

(a) $(n + 1)2^n$ (b) $2^n(n + 2)$

(c) $(n + 2)2^{n-1}$ (d) $(n + 2)2^{n+1}$

18. If A and B are two square matrices of the same order and m is a positive integer, then

$(A + B)^m = {}^mC_0A^m + {}^mC_1A^{m-1}B + {}^mC_2A^{m-2}B^2 + ... + {}^mC_mB^m$, if

(a) $AB = -BA$ (b) $A^m = 0, B^m = 0$

(c) $AB = 2BA$ (d) $AB = BA$

19. Let $I=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$ and $P=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-2\end{pmatrix}$. Then the matrix $P^3 + 2P^2 =$

(a) P (b) $I - P$

(c) $2I + P$ (d) $2I - P$

20. If t_5, t_{10}, t_{25} are 5^{th}, 10^{th} and 25^{th} terms of an A.P. respectively, then the value of $\begin{vmatrix}t_5 & t_{10} & t_{25}\\5&10&25\\1&1&1\end{vmatrix}$ is equal to

(a) -40 (b) 1

(c) $(\vec{a}\times\vec{b})^2$ (d) 0

21. Let S be the set of all real numbers. A relation R has been defined on S by $aRb \Rightarrow |a - b| \le 1$, then R is

(a) symmetric and transitive but not reflexive

(b) reflexive and transitive but not symmetric

(c) reflexive and symmetric but not transitive

(d) an equivalence relation

22. Let $f: N \to N$ defined by

$$f(n)=\begin{cases}\frac{n+1}{2}, & \text{if } n \text{ is odd}\\ \frac{n}{2}, & \text{if } n \text{ is even}\end{cases}$$, then f is

(a) onto but not one-one

(b) one-one and onto

(c) neither one-one nor onto

(d) one-one but not onto

23. The number of students who take both the subjects mathematics and chemistry is 30. This represents 10% of the enrolment in mathematics and 12% of the enrolment in chemistry. How many students take at least one of this two subjects?

(a) 520 (b) 490 (c) 560 (d) 480

24. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N$ are relatively prime, then

(a) $b = cd$ (b) $c = bd$

(c) $d = bc$ (d) none of these

25. A box contains 100 bulbs, out of which 10 are defective. A sample of 5 bulbs is drawn. The probability that none is defective is

(a) $\frac{9}{10}$ (b) $\left(\frac{1}{10}\right)^5$ (c) $\left(\frac{9}{10}\right)^5$ (d) $\left(\frac{1}{2}\right)^5$

26. Let A and B are two events such that $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, if $0 < P(A) < 1$ and $0 < P(B) < 1$, then $P(A \cup B)' =$

(a) $1 - P(A)$ (b) $1 - P(A')$

(c) $1 - P(A)P(B)$ (d) $(1 - P(A))P(B')$

27. A and B are two events such that $P(A) \ne 0$, $P(B/A)$ if

I. A is a subset of B

II. $A \cap B = \phi$ are respectively

(a) 1, 1 (b) 0, 1 (c) 0, 0 (d) 1, 0

28. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is

(a) 9 (b) 4 (c) 3 (d) 2

29. If $x=\frac{2\sin\theta}{1+\sin\theta+\cos\theta}$, then $\frac{1-\cos\theta+\sin\theta}{1+\sin\theta}$ is equal to

(a) x (b) $1/x$ (c) $1 + x$ (d) $1 - x$

30. If $\theta+\phi=\frac{\pi}{6}$, then the value of $(\sqrt{3}+\tan\theta)(\sqrt{3}+\tan\phi)$ is

(a) 4 (b) -4 (c) -1 (d) 1

31. The quadratic equation whose roots are $\sin18°$ and $\cos36°$ is

(a) $2x^2-\sqrt{5}x+1=0$ (b) $4x^2-2\sqrt{5}x+1=0$

(c) $2x^2+\sqrt{5}x+1=0$ (d) $4x^2+2\sqrt{5}x+1=0$

32. The smallest positive value of p for which the equation $\cos(p \sin x) = \sin(p \cos x)$ has a solution in $0 \le x \le 2\pi$, is

(a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{2\sqrt{2}}$ (d) $\frac{\pi}{4}$

33. If $\sin^{-1}x < \cos^{-1}x$, then

(a) $-1 \le x < \frac{1}{\sqrt{2}}$ (b) $0 \le x < \frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}} < x \le 1$ (d) $-1 < x < 0$

34. If two sides of a triangle are $2\sqrt{3}-2$ and $2\sqrt{3}+2$ and their included angle is 60°, then the other angles are

(a) 75°, 45° (b) 105°, 15°
(c) 60°, 60° (d) 90°, 30°

35. The circumcentre of a triangle with vertices (8, 6), (8, –2), (2, –2) is at the point

(a) (2, –1) (b) (1, –2) (c) (5, 2) (d) (2, 5)

36. If p and q are respectively the perpendiculars from the origin upon the straight lines whose equations are $x \sec\theta + y \operatorname{cosec}\theta = a$ and $x \cos\theta - y \sin\theta = a \cos 2\theta$, then $4p^2 + q^2$ is equal to

(a) $5a^2$ (b) $4a^2$ (c) $3a^2$ (d) a^2

37. A circle of radius $\sqrt{8}$ is passing through origin and the point (4, 0). If the centre lies on the line $y = x$, then the equation of the circle is

(a) $(x-2)^2 + (y-2)^2 = 8$
(b) $(x+2)^2 + (y+2)^2 = 8$
(c) $(x-3)^2 + (y-3)^2 = 8$
(d) $(x+3)^2 + (y+3)^2 = 8$

38. The area of a triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of latus rectum is (in sq. units)

(a) 20 (b) 18 (c) 17 (d) 19

39. The length of latus rectum of an ellipse is 4 units and distance between a focus and its nearest vertex is 3/2 units, the eccentricity is

(a) 1/2 (b) 1/3 (c) $1/\sqrt{2}$ (d) 1/4

40. $P(a\sec\theta, b\tan\theta)$ and $Q(-a\sec\theta, b\tan\theta)$ are two points on a hyperbola whose centre is at $O(0, 0)$ and eccentricity e. If PQO be an equilateral triangle, then

(a) $1 < e < 2$ (b) $e > 2$
(c) $e = \sqrt{2}$ (d) none of these

41. If $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$, $|\vec{b}| = 5$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then the area of the triangle formed by these two vector as two sides is

(a) $\frac{15}{4}$ (b) $\frac{15}{2}$ (c) 15 (d) $\frac{15\sqrt{3}}{2}$

42. Equation of the straight line through the point (2, 3, 1) and parallel to the line of intersection of the planes $x - 2y - z + 5 = 0$ and $x + y + 3z = 6$ is

(a) $\frac{x-2}{-5} = \frac{y-3}{-4} = \frac{z-1}{3}$ (b) $\frac{x-2}{5} = \frac{y-3}{-4} = \frac{z-1}{3}$

(c) $\frac{x-2}{4} = \frac{y-3}{3} = \frac{z-1}{2}$ (d) $\frac{x-2}{5} = \frac{y-3}{4} = \frac{z-1}{3}$

43. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-zero vectors such that each one of them being perpendicular to the sum of the other two vectors, then the value of $|\vec{a}+\vec{b}+\vec{c}|^2$ is

(a) $|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$ (b) $|\vec{a}| + |\vec{b}| + |\vec{c}|$

(c) $2(|\vec{a}| + |\vec{b}| + |\vec{c}|)$ (d) $\frac{1}{2}(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)$

44. The angle between two diagonals of a cube is

(a) $\cos^{-1}\left(\frac{1}{3}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c) 30° (d) 45°

45. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if

(a) $k = 2$ (b) $k = 0$ (c) $k = 3$ (d) $k = -1$

46. The value of $[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}]$ is equal to

(a) 0 (b) 1
(c) $2[\vec{a}\ \vec{b}\ \vec{c}]$ (d) 2

47. The domain of $y = \frac{1}{\sqrt{|x|-x}}$ is

(a) $0 \le x < \infty$ (b) $-\infty < x < 0$
(c) $-\infty < x \le 0$ (d) $1 \le x < 0$

48. If $f(x) = \frac{2^x - 1}{\sqrt{1+ax}-1}$ and $\lim_{x\to 0} f(x) = \log_e 4$, then a is equal to

(a) $\log_e 2$ (b) 1
(c) $\log_e 2 - 1$ (d) $\log_e 2 + 1$

49. The value of $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4} =$

(a) 0 (b) 1 (c) e^4 (d) e^5

50. If the function

$$f(x)=\begin{cases}\dfrac{4\sin(x^2)}{x}+2e^{-2x}+3, & x\neq 0\\ k, & x=0\end{cases}$$ is continuous at $x = 0$, then the value of k is

(a) 9 (b) 1 (c) 3 (d) 5

51. A function $f(x)$ is defined as follows for real x.

$$f(x)=\begin{cases}1-x^2, & \text{for } x<1\\ 0, & \text{for } x=1\\ 1+x^2, & \text{for } x>1\end{cases}$$ then

(a) $f(x)$ is not continuous at $x = 1$
(b) $f(x)$ is continuous but not differentiable at $x = 1$
(c) $f(x)$ is both continuous and differentiable at $x = 1$
(d) $f(x)$ is continuous everywhere but differentiable nowhere

52. If $f'(2)=1$, then $\lim\limits_{h\to 0}\dfrac{f(2+h^2)-f(2-h^2)}{2h^2}=$

(a) 0 (b) 1 (c) 2 (d) 1/2

53. If $y = (1 + x)(1 + x^2)(1 + x^4) \ldots (1 + x^{2n})$, then the value of $\left(\dfrac{dy}{dx}\right)_{x=0}$ is

(a) 0 (b) −1 (c) 1 (d) 2

54. The value of $\dfrac{dy}{dx}$ at $x=\dfrac{\pi}{2}$, where y is given by $y=x^{\sin x}+\sqrt{x}$, is

(a) $1+\dfrac{1}{\sqrt{2\pi}}$ (b) 1

(c) $\dfrac{1}{\sqrt{2\pi}}$ (d) $1-\dfrac{1}{\sqrt{2\pi}}$

55. The second order derivative of $a\sin^3 t$ with respect to $a\cos^3 t$ at $t = \pi/4$ is

(a) 2 (b) $\dfrac{1}{12a}$ (c) $\dfrac{4\sqrt{2}}{3a}$ (d) $\dfrac{3a}{4\sqrt{2}}$

56. If $y = \sin(2\sin^{-1}x)$, then it satisfies the differential equation

(a) $(1 - x^2)y_2 - xy_1 + 4y = 0$
(b) $(1 + x^2)y_2 - xy_1 + 4y = 0$
(c) $(1 - x^2)y_2 - xy_1 + y = 0$
(d) $(1 + x^2)y_2 - xy_1 + y = 0$

57. If $x^2 + y^2 = 1$, then

(a) $yy' - (2y')^2 + 1 = 0$ (b) $yy'' + (y')^2 + 1 = 0$
(c) $yy'' - (y')^2 - 1 = 0$ (d) $yy''- (2y')^2 + 1 = 0$

58. If the function $f(x)$ satisfies $\lim\limits_{x\to 1}\dfrac{f(x)-2}{x^2-1}=\pi$, then $\lim\limits_{x\to 1} f(x)=$

(a) 1 (b) 2 (c) 0 (d) 3

59. If the function $f(x)$ defined by $f(x)=\dfrac{x^{100}}{100}+\dfrac{x^{99}}{99}+\ldots+\dfrac{x^2}{2}+x+1$, then $f'(0) =$

(a) $100f'(0)$ (b) 100
(c) 1 (d) −1

60. Let $f(x)$ be differentiable function and $f'(4) = 5$, then $\lim\limits_{x\to 2}\dfrac{f(4)-f(x^2)}{x-2}$ equals

(a) 0 (b) 5 (c) 20 (d) −20

CATEGORY-2

For each correct answer 2 marks will be allotted and negative marking of 25% of total marks (1/2) will be implemented for wrong answers. For marking more than 1 answer, marks will be deducted.

61. A particle moves along x-axis according to the equation $x=\dfrac{1}{3}t^3-\dfrac{1}{2}\cos 2t+3.5$. The distance travelled by the particle between $t = 0$ and $t = \pi/2$ is

(a) 1 (b) 2
(c) $\dfrac{\pi^2}{24}+1$ (d) $\dfrac{\pi^3}{24}+1$

62. If $f(x) = \sin x - bx + c$ decreases for all real values of x, then

(a) $b < 0$ (b) $b = 0$
(c) $0 < b < 1$ (d) $b > 1$

63. If the line $ax + by + c = 0$ is a tangent to the curve $xy = 4$, then

(a) $a < 0, b > 0$ (b) $a \le 0, b > 0$
(c) $a < 0, b < 0$ (d) $a \le 0, b < 0$

64. If $f(x)=\int_1^x t(t-3)(2t-3)^2\,dt$, then the function $f(x)$ will be minimum at

(a) $x = 0$ (b) $x = 3$
(c) $x = 3/2$ (d) none of these

65. $\int\dfrac{x+1}{x(xe^x+1)^2}\,dx$ is equal to

(a) $\log\left|\dfrac{xe^x}{1+e^x}\right|-\dfrac{1}{1+xe^x}+c$

(b) $\log\left|\dfrac{1+xe^x}{xe^x}\right|+\dfrac{1}{1+xe^x}+c$

(c) $\log\left|\dfrac{xe^x}{1+xe^x}\right|+\dfrac{1}{1+xe^x}+c$

(d) none of these

66. $\int\left\{\frac{\log x-1}{1+(\log x)^2}\right\}^2 dx =$

(a) $\frac{\log x}{(\log x)^2+1}+c$ (b) $\frac{x}{x^2+1}+c$

(c) $\frac{xe^x}{x^2+1}+c$ (d) $\frac{x}{(\log x)^2+1}+c$

67. If $f(x)$ is a differentiable function and $\int_0^{t^2} x f(x)dx=\frac{2}{5}t^5$, then the value of $f\left(\frac{4}{25}\right)$ is

(a) 2/5 (b) 5/2 (c) 1 (d) –5/2

68. $\int_2^4 \log[x]\, dx =$

(a) log 2 (b) log 3

(c) log 5 (d) log 6

69. If $f(x) = \max\{\sin x, \cos x\}$, then the value of $\int_{\pi/6}^{\pi/3} f(x)dx$ is

(a) 0 (b) $\sqrt{2}-1$

(c) $\sqrt{2}$ (d) $\sqrt{2}+1$

70. The differential equation of all circles passing through the origin and having their centres on the x-axis, is

(a) $y^2=x^2+2xy\frac{dy}{dx}$ (b) $y^2=x^2-2xy\frac{dy}{dx}$

(c) $x^2=y^2+xy\frac{dy}{dx}$ (d) $x^2=y^2+3xy\frac{dy}{dx}$

CATEGORY-3

In this section more than 1 answer can be correct. Candidates will have to mark all the correct answers, for which 2 marks will be awarded. If, candidates marks one correct and one incorrect answer then no marks will be awarded. But if, candidate marks only correct, without marking any incorrect, formula below will be used to allot marks.

2 × (no. of correct response/total no. of correct options)

71. Let $f_n(\theta)=\tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)....(1+\sec 2^n\theta)$, then

(a) $f_5\left(\frac{\pi}{128}\right)=1$ (b) $f_3\left(\frac{\pi}{32}\right)=1$

(c) $f_4\left(\frac{\pi}{64}\right)=1$ (d) $f_2\left(\frac{\pi}{16}\right)=1$

72. If $5x - y$, $2x + y$, $x + 2y$ are in A.P. and $(x - y)^2$, $(xy + 1)$, $(y + 1)^2$ are in G.P., $x \neq 0$, then $x + y =$

(a) 3/4 (b) 3 (c) –5 (d) –6

73. The point P divides the line segment joining the points $A(-5, 1)$ and $B(3, 5)$ in $\lambda : 1$. The coordinates of the points Q and R are respectively (1, 5) and (7, 2). If the area of the triangle PQR is 2 square units, then the value of λ is

(a) 19/5 (b) 31/9 (c) 23 (d) 19

74. Let A and A' are the vertices of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose foci is S and S' and eccentricity e. Let P is a point on the ellipse such that $\frac{ar(\Delta AA'P)}{ar(\Delta SS'P)}=K$, then

(a) $0 < K < 1$ (b) $K > 1$

(c) K, is only dependent on e

(d) K, is only dependent on b and e

75. If $f(x)$ is a differentiable function in [0, 1] and $f(0) = f(1) = 0$, then $f'(c) + \alpha f(c) = 0$ for $\alpha =$

(a) –1 (b) 1 (c) 0 (d) 2

76. $\int\sqrt{1+\sin 2x}\, dx =$

(a) $\sin x+\cos x+c\ \forall\ x\in R$

(b) $\sin x-\cos x+c\ \forall\ x\in R$

(c) $\sin x-\cos x+c,\ x\in\left[\frac{-\pi}{4},\frac{3\pi}{4}\right]$

(d) $\cos x-\sin x+c,\ x\in\left[\frac{3\pi}{4},\frac{7\pi}{4}\right]$

77. $f(x)=\int_{-1}^{x} t(e^t-1)(t-1)(t-2)^3(t-3)^5 dt$ has a local minima at $x =$

(a) 0 (b) 1 (c) 2 (d) 3

78. The area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is

(a) $e - 1$ (b) $\int_1^e \ln(e+1-y)dy$

(c) $e-\int_1^e e^x dx$ (d) $\int_1^x \ln y dx$

79. If $|\vec{a}|=4$, $|\vec{b}|=2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$, then $(\vec{a}\times\vec{b})^2$ is equal to

(a) 48 (b) 16

(c) $|\vec{a}|^2$ (d) none of these

80. If A and B are two events such that $P(A\cup B)\geq\frac{3}{4}$ and $\frac{1}{8}\leq P(A\cap B)\leq\frac{3}{8}$. Then

(a) $P(A)+P(B)\leq\frac{11}{8}$ (b) $P(A)\cdot P(B)\leq\frac{3}{8}$

(c) $P(A)+P(B)\geq\frac{7}{8}$ (d) none of these

SOLUTIONS

1. (c) : We have, $a = 2$ and $S_5 = 5$

$\therefore\ S_5 = \frac{5}{2}[(2\times 2)+(5-1)d]$

$\Rightarrow\ \frac{5}{2}[4+4d]=5\ \Rightarrow\ 2+2d=1$

$\Rightarrow\ 2d = -1\ \Rightarrow\ d = -1/2$

Now, $a_{31} = a + (n-1)d = 2+30\times\left(-\frac{1}{2}\right)=-13$

2. (d) : Given that $ar = 24$ and $ar^4 = 3$

$\therefore\ \frac{ar^4}{ar}=\frac{3}{24}\ \Rightarrow\ r^3=\frac{1}{8}=\left(\frac{1}{2}\right)^3\ \therefore\ r=\frac{1}{2}$

$S_6=\frac{48\left(1-\frac{1}{2^6}\right)}{1-\frac{1}{2}}=\frac{48\times 63}{64\times\frac{1}{2}}=\frac{189}{2}$

3. (d) : Given that $a:b=\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$

Let $a=(3+2\sqrt{2})k$ and $b=(3-2\sqrt{2})k$

$\therefore\ \frac{a+b}{2}=3k$ and $ab=k^2$

Now, $\frac{\frac{a+b}{2}}{\sqrt{ab}}=\frac{3}{1}$

Thus, the required ratio is 3 : 1.

4. (d) : Given that $S_\infty = 4\ \Rightarrow\ \frac{a}{1-r}=4$

$\Rightarrow\ a + 4r = 4$ ...(i)

Also, given that $ar=\frac{3}{4}\ \Rightarrow\ r=\frac{3}{4a}$ ...(ii)

From (i) and (ii), we get $a+\frac{3}{a}=4$

$\Rightarrow\ a^2 - 4a + 3 = 0\ \Rightarrow\ (a-3)(a-1) = 0$

$\Rightarrow\ a = 3, r = 1/4$ or $a = 1, r = 3/4$

5. (a) : Given that $\log_e 5$, $\log_e(5^x - 1)$, $\log_e\left(5^x-\frac{11}{5}\right)$ are in A.P.

$\therefore\ 2\log_e(5^x-1)=\log_e 5+\log_e\left(5^x-\frac{11}{5}\right)$

$\Rightarrow\ (5^x-1)^2=5\left(5^x-\frac{11}{5}\right)$

$\Rightarrow\ (5^x)^2 - 2\cdot 5^x + 1 = 5\cdot 5^x - 11$

$\Rightarrow\ (5^x)^2 - 7\cdot 5^x + 12 = 0$

$\Rightarrow\ (5^x - 3)(5^x - 4) = 0$

$\Rightarrow\ 5^x - 3 = 0\ \Rightarrow\ 5^x = 3\ \therefore\ x = \log_5 3$

and $5^x - 4 = 0\ \Rightarrow\ 5^x = 4\ \therefore\ x = \log_5 4$

6. (a) : By verification, $x = 1, 2$

7. (a) : $(z_3 - z_1) = (z_2 - z_1)(\cos 90° + i\sin 90°)$

$\Rightarrow\ z_3 - z_1 = i(z_2 - z_1)$

$\Rightarrow\ (z_3 - z_1)^2 = -(z_2 - z_1)^2$

$\Rightarrow\ (z_3 - z_1)^2 + (z_2 - z_1)^2 = 0$

8. (a) : $|x_1z_1 - y_1z_2|^2 + |y_1z_1 + x_1z_2|^2$

$= |x_1z_1|^2 + |y_1z_2|^2 - 2Re(x_1y_1z_1z_2) + |y_1z_1|^2 + |x_1z_2|^2 + 2Re(x_1y_1z_1z_2)$

$= x_1^2|z_1|^2 + y_1^2|z_2|^2 + y_1^2|z_1|^2 + x_1^2|z_2|^2$

$= 2(x_1^2 + y_1^2)4^2 = 32(x_1^2 + y_1^2)$

9. (b) : Given that α and β are the roots of the equation $x^2 - ax + b^2 = 0$

$\therefore\ \alpha + \beta = a$ and $\alpha\beta = b^2$

$\therefore\ \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = a^2 - 2b^2$

10. (d) : Let α, β be the roots of the equation $2x^2 + 3x + 1 = 0$

$\therefore\ \alpha + \beta = -3/2$ and $\alpha\beta = 1/2$

Now, $\alpha^2+\beta^2=\frac{9}{4}-1=\frac{5}{4}$ and $\alpha^2\beta^2=\frac{1}{4}$

$\therefore$ The required equation is

$x^2-\frac{5}{4}x+\frac{1}{4}=0\ \Rightarrow\ 4x^2-5x+1=0$

11. (d) : T_n : The number of triangles formed by joining the vertices of a regular polygon of n sides

$T_{n+1} - T_n = 36$ (given)

${}^{n+1}C_3 - {}^nC_3 = 36\ \Rightarrow\ {}^nC_2 = 36$

$\Rightarrow\ n(n-1) = 72\ \Rightarrow\ n^2 - n - 72 = 0$

$\Rightarrow\ (n-9)(n+8) = 0\ \Rightarrow\ n = 9\ (\because n \neq -8)$

12. (b) : There are 10 digits 0 to 9. Now keeping fix 67 at the beginning, the remaining 3 places of 5 digit telephone number may be arranged from 8 digits in ${}^8P_3$ ways. Thus the number of telephone numbers which can be constructed $= {}^8P_3 = 336$.

13. (d) : $1^2+2^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$

$=\frac{n(n+1)\left(n+\frac{1}{2}\right)}{3}$

$>\frac{n\cdot n\cdot n}{3}\ \left[\because n+1>n \text{ and } n+\frac{1}{2}>n\right]$

$>\frac{n^3}{3}$

14. (a) : $1! + 2! + 3! + 4! + \ldots + 11!$

$= 1 + 2 + 6 + 24 + 120 + \ldots$

$= 9 + 12(2 + 3 + \ldots.)$

$= 9 + 12k$ [$\because$ after first 3 terms all are divisible by 12]

$\therefore$ Remainder = 9

15. (c) : The given expansion is $(1 + x)^{44}$

Now $t_{20+1} = {}^{44}C_{20}\, x^{20}$

and $t_{21+1} = {}^{44}C_{21}\, x^{21}$

By the problem, we have

$${}^{44}C_{21}\, x^{21} = {}^{44}C_{20}\, x^{20}$$

$$\Rightarrow \frac{\lfloor 44}{\lfloor 21\ \lfloor 23}x = \frac{\lfloor 44}{\lfloor 20\ \lfloor 24} \Rightarrow \frac{x}{21\lfloor 20 \times \lfloor 23} = \frac{1}{\lfloor 20 \times 24 \lfloor 23}$$

$$\Rightarrow \frac{x}{21} = \frac{1}{24} \Rightarrow x = \frac{21}{24} = \frac{7}{8}$$

16. (a) : The given expansion is $\left(\frac{10}{x} + \frac{x}{10}\right)^{10}$

The middle term is $\left(\frac{10}{2} + 1\right)^{\text{th}}$ term *i.e.* 6^{th} term

$$\therefore\ t_{5+1} = {}^{10}C_5 \left(\frac{10}{x}\right)^5 \left(\frac{x}{10}\right)^5 = {}^{10}C_5$$

17. (c) : ${}^nC_0 + 2{}^nC_1 + 3{}^nC_2 + ... + (n + 1){}^nC_n$

$= ({}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n) + ({}^nC_1 + 2{}^nC_2 + 3{}^nC_3 + ... + n\,{}^nC_n)$

$$= 2^n + \left(n + \frac{2n(n-1)}{2!} + ... + n\right)$$

$$= 2^n + n\left(1 + (n-1) + \frac{(n-1)(n-2)}{2!} + ... + 1\right)$$

$$= 2^n + n(1+1)^{n-1} = 2^n + n \cdot \frac{2^n}{2}$$

$$= 2^n\left(1 + \frac{n}{2}\right) = (n+2)2^{n-1}$$

18. (d) : $(A + B)^m = {}^mC_0\, A^m + {}^mC_1 A^{m-1} B + {}^mC_2 A^{m-2} B^2 + ... + {}^mC_m\, B^m$

Putting $m = 2$, we get

$$(A + B)^2 = {}^2C_0 A^2 + {}^2C_1 AB + {}^2C_2 B^2$$

$$\Rightarrow (A + B)^2 = A^2 + 2AB + B^2 \quad ...(i)$$

But $(A + B)^2 = (A + B)(A + B)$

$$= A(A + B) + B(A + B)$$

$$\Rightarrow (A + B)^2 = A^2 + B^2 + AB + BA \quad ...(ii)$$

From (i) and (ii), we get

$$A^2 + 2AB + B^2 = A^2 + B^2 + AB + BA$$

$$\Rightarrow AB = BA$$

19. (c) : $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

$$\therefore\ P^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix},$$

$$P^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -8 \end{pmatrix}$$

$$\therefore\ P^3 + 2P^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -8 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = P + 2I$$

20. (d) : $t_5 = a + 4d$, $t_{10} = a + 9d$ and $t_{25} = a + 24d$

Now, $\begin{vmatrix} t_5 & t_{10} & t_{25} \\ 5 & 10 & 25 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} a+4d & a+9d & a+24d \\ 5 & 10 & 25 \\ 1 & 1 & 1 \end{vmatrix}$

Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} -5d & -15d & a+24d \\ -5 & -15 & 25 \\ 0 & 0 & 1 \end{vmatrix} = 1(75d - 75d) = 0$$

21. (c) : For reflexive : Let $a \in S$

then $|a - a| = 0 < 1 \Rightarrow a\ R\ a$

So, R is reflexive.

For symmetric : Let $aRb \Rightarrow |a - b| \le 1$

$$\Leftrightarrow |-(b - a)| \le 1$$

$$\Leftrightarrow |b - a| \le 1$$

$$\Leftrightarrow bRa$$

So, R is symmetric.

For transitive : Let $1, 2, 3 \in S$

Then, $|1 - 2| = 1 \le 1 \Rightarrow 1R2$

and $|2 - 3| = 1 \le 1 \Rightarrow 2R3$

But $|1 - 3| = 2 > 1$

So, R is not transitive.

22. (a) : We have, $f(1) = \frac{1+1}{2} = \frac{2}{2} = 1$

$$f(2) = \frac{2}{2} = 1$$

Thus, $f(1) = f(2)$ but $1 \ne 2$

$\therefore$ $f(n)$ is not one-one.

In order to find that f is onto or not, consider element $n \in N$

If n is odd, then $(2n - 1)$ is also odd and

$$f(2n-1) = \frac{2n-1+1}{2} = \frac{2n}{2} = n$$

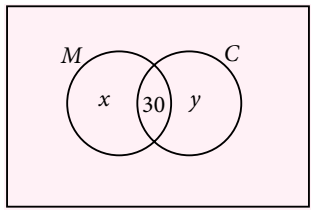
If n is even, then $2n$ is also even and

$$f(2n)=\frac{2n}{2}=n$$

Thus, for each $n \in N$, there exists its pre-image in N.

$\therefore$ f is onto.

23. (a) : Let x be the number of students who take only mathematics and y be the number of students who take only chemistry.



$$\therefore \quad \frac{10}{100}(x+30)=30 \implies x=270$$

Similarly $\frac{12}{100}(y+30)=30 \implies y=220$

Number of students who take at least one of these two subjects = 270 + 220 + 30 = 520

24. (c) : Given that $aN = \{ax : x \in N\}$

$\therefore$ $bN = \{bx : x \in N\}$, $cN = \{cx : x \in N\}$

and $dN = \{dx : x \in N\}$

As $bN \cap cN = dN$

$\Rightarrow$ $d = dx_1 \in bN \cap cN$

$\Rightarrow$ $d = bx_1$ and $d = cx_2$, where $x_1, x_2 \in N$

$\Rightarrow$ $\frac{d}{b}$ and $\frac{d}{c}$ are natural numbers

$\Rightarrow$ $\frac{d}{bc} \in N$, because b and c are relatively prime. …(i)

Also $bc \in bN$ and $bc = cb \in cN$

$\therefore$ $bc \in bN \cap cN$ or $bc \in dN$

$\Rightarrow$ $bc = dx_3$ for $x_3 \in N \implies \frac{bc}{d} \in N$ …(ii)

From (i) and (ii), $bc = d$

25. (c) : Total number of bulbs = 100

Probability of defective bulbs $(p) = \frac{10}{100}=\frac{1}{10}$

$\therefore$ Probability of non-defective bulbs $(q) = \frac{9}{10}$

So, $P(X = x) = {}^nC_x\, p^x\, q^{n-x} = {}^5C_0\left(\frac{9}{10}\right)^5=\left(\frac{9}{10}\right)^5$

26. (d) : $P(A \cup B)' = P(A' \cap B') = P(A') \cdot P(B')$

$= (1 - P(A))\, P(B')$

27. (d) : (i) It is given that, $A \subset B \implies A \cap B = A$

$\therefore$ $P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{P(A)}{P(A)}=1$

(ii) If $A \cap B = \phi$, then $P(B/A)=\frac{P(A\cap B)}{P(A)}=0$

28. (d) : Given that $\sum_{i=1}^{9}(x_i-5)=9$

$\Rightarrow$ $\sum_{i=1}^{9}x_i-45=9$

$\therefore$ $\sum_{i=1}^{9}x_i=54 \implies \frac{1}{9}\sum_{i=1}^{9}x_i=6$

Now, $\sum_{i=1}^{9}(x_i-5)^2=45$

$\Rightarrow$ $\frac{1}{9}\sum_{i=1}^{9}x_i^2-10\cdot\frac{1}{9}\sum_{i=1}^{9}x_i+25=5$

$\Rightarrow$ $\frac{1}{9}\sum_{i=1}^{9}x_i^2=65-25=40$

$\therefore$ $S.D.=\sqrt{\frac{1}{9}\sum_{i=1}^{9}x_i^2-\left(\frac{1}{9}\sum_{i=1}^{9}x_i\right)^2}$

$=\sqrt{40-36}=\sqrt{4}=2$

29. (a) : $x=\frac{2\sin\theta}{1+\sin\theta+\cos\theta}$

$=\frac{2\sin\theta\{1-(\sin\theta+\cos\theta)\}}{1-(\sin\theta+\cos\theta)^2}=\frac{2\sin\theta\{1-(\sin\theta+\cos\theta)\}}{-2\sin\theta\cos\theta}$

$=\frac{1-(\sin\theta+\cos\theta)}{-\cos\theta}=\frac{(1-\sin\theta-\cos\theta)(1+\sin\theta)}{-\cos\theta(1+\sin\theta)}$

$=\frac{1-\sin^2\theta-\cos\theta(1+\sin\theta)}{-\cos\theta(1+\sin\theta)}=\frac{\cos^2\theta-\cos\theta(1+\sin\theta)}{-\cos\theta(1+\sin\theta)}$

$=\frac{\cos\theta-1-\sin\theta}{-(1+\sin\theta)}$ $\therefore$ $x=\frac{1-\cos\theta+\sin\theta}{1+\sin\theta}$

30. (a) : Given that $\theta+\phi=\frac{\pi}{6}$

$\therefore$ $\tan(\theta+\phi)=\tan(\pi/6)$

$\Rightarrow$ $\sqrt{3}(\tan\theta+\tan\phi)=1-\tan\theta\tan\phi$

$\Rightarrow$ $\sqrt{3}\tan\theta+\sqrt{3}\tan\phi+\tan\theta\tan\phi=1$

$\Rightarrow$ $3+\sqrt{3}\tan\theta+\sqrt{3}\tan\phi+\tan\theta\tan\phi=4$

$\Rightarrow$ $\sqrt{3}(\sqrt{3}+\tan\phi)+\tan\theta(\sqrt{3}+\tan\phi)=4$

$\Rightarrow$ $(\sqrt{3}+\tan\theta)(\sqrt{3}+\tan\phi)=4$

31. (b) : Let $\alpha = \sin 18° = \frac{\sqrt{5}-1}{4}$

and $\beta = \cos 36° = \frac{\sqrt{5}+1}{4}$

Now $\alpha+\beta = \frac{\sqrt{5}-1}{4}+\frac{\sqrt{5}+1}{4} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}$

$\alpha\beta = \frac{\sqrt{5}-1}{4}\times\frac{\sqrt{5}+1}{4} = \frac{5-1}{16} = \frac{1}{4}$

Thus the required equation is

$$x^2 - \frac{\sqrt{5}}{2}x + \frac{1}{4} = 0 \Rightarrow 4x^2 - 2\sqrt{5}x + 1 = 0$$

32. (c) : $\cos(p\sin x) = \sin(p\cos x) = \cos\left(\frac{\pi}{2} - p\cos x\right)$

$\therefore \quad p\sin x = \frac{\pi}{2} - p\cos x \Rightarrow p(\sin x + \cos x) = \frac{\pi}{2}$

$\Rightarrow p\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right) = \frac{\pi}{2\sqrt{2}}$

$\Rightarrow p\cos\left(x - \frac{\pi}{4}\right) = \frac{\pi}{2\sqrt{2}}$

$\Rightarrow p = \frac{\pi}{2\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)}$

Clearly, the value of p will be positive and least when the value of $\cos\left(x-\frac{\pi}{4}\right)$ is positive and greatest *i.e.* when $\cos\left(x-\frac{\pi}{4}\right) = 1$

$\therefore$ The required value of p is $\frac{\pi}{2\sqrt{2}}$.

33. (a) : $\because \quad -\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2} \quad \therefore \quad -1 \le x \le 1;$

Now given that $\sin^{-1}x < \cos^{-1}x$

$\Rightarrow 2\sin^{-1}x < \sin^{-1}x + \cos^{-1}x$

$\Rightarrow 2\sin^{-1}x < \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x < \frac{\pi}{4} \Rightarrow \sin(\sin^{-1}x) < \sin\frac{\pi}{4}$

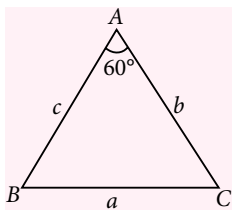
$\Rightarrow x < \frac{1}{\sqrt{2}}. \quad \therefore \quad -1 \le x < \frac{1}{\sqrt{2}}$

34. (b) : Let $b = 2\sqrt{3}+2$

$c = 2\sqrt{3}-2$

and $\angle A = 60°$

$\therefore \quad \tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2}$



$= \frac{4}{4\sqrt{3}}\cot 30° = 1$

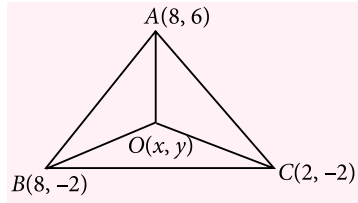
$\tan\frac{B-C}{2} = \tan 45° \quad \therefore \quad B - C = 90°$ ...(i)

But $A + B + C = 180°$

$\Rightarrow \quad B + C = 120° \quad [\because \angle A = 60°]$ ...(ii)

From (i) and (ii), we get $\angle B = 105°$ and $\angle C = 15°$

35. (c) :



Let the circumcentre be $O(x, y)$

$\therefore \quad \overrightarrow{OA} = \sqrt{(x-8)^2 + (y-6)^2}$

$\overrightarrow{OB} = \sqrt{(x-8)^2 + (y+2)^2}$

$\overrightarrow{OC} = \sqrt{(x-2)^2 + (y+2)^2}$

Here, $\overrightarrow{OA} = \overrightarrow{OB} = \overrightarrow{OC}$

Now, $OA = OB$ gives $(y - 6)^2 = (y + 2^2)^2$

$\Rightarrow \quad y^2 - 12y + 36 = y^2 + 4y + 4$

$\Rightarrow \quad 16y = 32 \quad \therefore \quad y = 2$

and $\overrightarrow{OB} = \overrightarrow{OC}$ gives $x^2 - 16x + 64 = x^2 - 4x + 4$

$\Rightarrow \quad 12x = 60 \quad \therefore \quad x = 5$

Thus circumcentre is (5, 2)

36. (d) : $p = \left|\frac{-a}{\sqrt{\sec^2\theta + \text{cosec}^2\theta}}\right|$ and $q = \left|\frac{-a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}}\right|$

$\Rightarrow \quad p^2 = \frac{a^2}{\sec^2\theta + \text{cosec}^2\theta}$ and $q^2 = \frac{a^2\cos^2 2\theta}{1}$

$p^2 = \frac{a^2}{4}\sin^2 2\theta$ and $q^2 = a^2\cos^2 2\theta$

$\therefore \quad 4p^2 + q^2 = a^2$

37. (a) : Let the required circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

$\therefore \quad \sqrt{g^2 + f^2 - c} = \sqrt{8} \Rightarrow g^2 + f^2 = 8$ ... (i)

$\Rightarrow \quad 16 + 8g = 0 \Rightarrow g = -2$

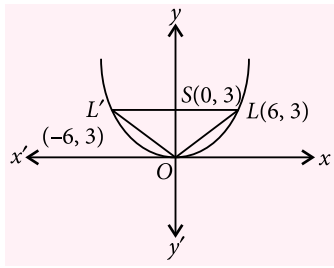
Putting the value of g in (i), we get

$f^2 = 4 \Rightarrow f = -2$

Required circle is $x^2 + y^2 - 4x - 4y = 0$

$\Rightarrow \quad (x - 2)^2 + (y - 2)^2 = 8$

38. (b) :



The given parabola is $x^2 = 12y$

$\Rightarrow \quad x^2 = 4 \cdot 3 \cdot y$

Thus from the diagram, the area of the triangle OLL' is

$$\frac{1}{2} \times 12 \times 3 \text{ sq. unit} = 18 \text{ sq. unit}$$

39. (b) : Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \therefore \quad \frac{2b^2}{a} = 4 \quad i.e. \quad b^2 = 2a \qquad \ldots(i)$$

Also, given that distance between focus and its nearest vertex is 3/2.

$$i.e., \quad a - ae = \frac{3}{2} \quad \Rightarrow \quad 1 - e = \frac{3}{2a}$$

$$\Rightarrow \quad e = 1 - \frac{3}{2a} = \frac{2a-3}{2a} \qquad \ldots(ii)$$

$$\Rightarrow \quad e^2 = \frac{4a^2 - 12a + 9}{4a^2} \quad \Rightarrow \quad 1 - \frac{b^2}{a^2} = \frac{4a^2 - 12a + 9}{4a^2}$$

$$\Rightarrow \quad 1 - \frac{2}{a} = \frac{4a^2 - 12a + 9}{4a^2} \quad [\text{by (i)}]$$

$\Rightarrow \quad 4a(a - 2) = 4a^2 - 12a + 9$

$\Rightarrow \quad 4a = 9 \quad \therefore \quad a = \frac{9}{4} \qquad \ldots(iii)$

From (ii) and (iii), we get $e = 1/3$.

40. (b) : $PQ = OP \quad \Rightarrow \quad PQ^2 = OP^2$.

$\therefore \quad 4a^2\sec^2\theta = a^2\sec^2\theta + b^2\tan^2\theta$

We have $\frac{b^2}{a^2} = 3\text{cosec}^2\theta \quad \Rightarrow \quad e^2 - 1 = 3\text{cosec}^2\theta$

$$\Rightarrow \quad \frac{e^2 - 1}{3} = \text{cosec}^2\theta > 1$$

41. (a) : Here $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$

$$\therefore \quad |\vec{a}| = \sqrt{1^2 + 2^2 + 2^2} = 3$$

We are given that $|\vec{b}| = 5$

$$\text{Required area} = \frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}|\vec{a}||\vec{b}|\sin\theta$$

$$= \frac{1}{2} \times 3 \times 5 \times \sin\frac{\pi}{6} = \frac{15}{4}$$

42. (a) : Let the d.r's of the line of intersection of the planes be $<a, b, c>$

$\therefore \quad a - 2b - c = 0 \qquad \ldots(i)$

and $a + b + 3c = 0 \qquad \ldots(ii)$

Solving (i) and (ii), we get

$$\frac{a}{-5} = \frac{b}{-4} = \frac{c}{3} = k \text{ (say)}$$

$\therefore \quad a = -5k, \; b = -4k, \; c = 3k$

$$\Rightarrow \quad \frac{x-2}{-5} = \frac{y-3}{-4} = \frac{z-1}{3}$$

43. (a) : $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$

$$+2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

Given that $\vec{a} \perp (\vec{b} + \vec{c}) \Rightarrow \vec{a} \cdot (\vec{b} + \vec{c}) = 0 \qquad \ldots(i)$

Similarly, $\vec{b} \perp (\vec{a} + \vec{c}) \Rightarrow \vec{b} \cdot (\vec{a} + \vec{c}) = 0 \qquad \ldots(ii)$

and $\vec{c} \perp (\vec{a} + \vec{b}) \Rightarrow \vec{c} \cdot (\vec{a} + \vec{b}) = 0 \qquad \ldots(iii)$

Adding (i), (ii) and (iii), we get

$$2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow \quad |\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$$

44. (a)

45. (b) : The given lines are

$$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k} \qquad \ldots(i)$$

and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1} \qquad \ldots(ii)$

Now, (i) and (ii) are coplanar.

$$\therefore \quad \begin{vmatrix} 1-2 & 4-3 & 5-4 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

On simplification, we get $k = 0$ or $k = -3$

46. (a)

47. (b) : $y = \frac{1}{\sqrt{|x| - x}}$

y will be defined if $|x| - x > 0$

$i.e. \; |x| > x \quad i.e. \quad x < 0$

48. (b) : $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \frac{2^x - 1}{\sqrt{1 + ax} - 1} = \lim\limits_{x \to 0} \frac{\frac{2^x - 1}{x}}{\frac{\sqrt{1 + ax} - 1}{x}}$

$$= \frac{\lim\limits_{x \to 0} \frac{2^x - 1}{x}}{\lim\limits_{x \to 0} \frac{\sqrt{1 + ax} - 1}{x}} = \frac{\log_e 2}{a \times \frac{1}{\sqrt{1 + 0} + 1}} = \frac{2}{a}\log 2$$

$\therefore \quad \frac{2}{a}\log_e 2 = \log_e 4 = 2\log_e 2 \quad \therefore \quad a = 1$

49. (d) : $\lim\limits_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4} = \lim\limits_{z\to 0}\left(\frac{1+6z}{1+z}\right)^{\frac{1}{z}+4}$ $\left[\text{Put } \frac{1}{x} = z\right]$

$= \lim\limits_{z\to 0}\left(\frac{1+6z}{1+z}\right)^{4} \cdot \lim\limits_{z\to 0}\left(\frac{1+6z}{1+z}\right)^{1/z} = 1 \cdot \frac{\lim\limits_{z\to 0}(1+6z)^{1/z}}{\lim\limits_{z\to 0}(1+z)^{1/z}}$

Now, $\lim\limits_{z\to 0}(1+6z)^{1/z} = \lim\limits_{u\to 0}(1+u)^{6/u}$ [Put $6z = u$]

$= \left[\lim\limits_{u\to 0}(1+u)^{1/u}\right]^6 = e^6$

$\therefore$ The given limit $= 1 \cdot \frac{e^6}{e} = e^5$

50. (d) : Since $f(x)$ is continuous at $x = 0$

$\therefore \quad k = \lim\limits_{x\to 0}\left[\frac{4\sin(x^2)}{x} + 2e^{-2x} + 3\right]$

$= 4\lim\limits_{x\to 0}\frac{x\sin(x^2)}{x^2} + 2\lim\limits_{x\to 0} e^{-2x} + 3$

$= 4 \cdot 0 \cdot 1 + 2 + 3 = 5$

51. (a) : $\lim\limits_{x\to 1^{-0}} f(x) \neq \lim\limits_{x\to 1^{+0}} f(x)$

52. (b) : $\lim\limits_{h\to 0}\frac{f(2+h^2) - f(2-h^2)}{2h^2}$

$= \lim\limits_{h\to 0}\frac{\{f(2+h^2) - f(2)\} - \{f(2-h^2) - f(2)\}}{2h^2}$

$= \frac{1}{2}\left[\lim\limits_{h^2\to 0}\frac{f(2+h^2) - f(2)}{h^2} + \lim\limits_{-h^2\to 0}\frac{f(2-h^2) - f(2)}{-h^2}\right]$

$= \frac{1}{2}\left[f'(2) + f'(2)\right] = \frac{1}{2}\cdot 2f'(2) = f'(2) = 1$

53. (c) : $y = (1 + x)(1 + x^2)(1 + x^4) \ldots\ldots (1 + x^{2n})$

$\therefore \quad \log y = \log(1 + x) + \log(1 + x^2) + \log(1 + x^4) + \ldots + \log(1 + x^{2n})$

$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \ldots + \frac{2nx^{2n-1}}{1+x^{2n}}\right]$

$\Rightarrow \quad \frac{dy}{dx} = (1+x)(1+x^2)(1+x^4)\ldots.(1+x^{2n})$

$\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \ldots. + \frac{2nx^{2n-1}}{1+x^{2n}}\right]$

$\therefore \quad \left(\frac{dy}{dx}\right)_{x=0} = 1$

54. (a) : $y = x^{\sin x} + \sqrt{x} = e^{\log_e x^{\sin x}} + \sqrt{x}$

$\Rightarrow \quad y = e^{\sin x \log x} + x^{1/2}$

$\Rightarrow \quad \frac{dy}{dx} = e^{\sin x \log x}\left(\frac{\sin x}{x} + \cos x \log x\right) + \frac{1}{2\sqrt{x}}$

$\Rightarrow \quad \frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right) + \frac{1}{2\sqrt{x}}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = \frac{\pi}{2}\left(\frac{2}{\pi} + 0\right) + \frac{1\times\sqrt{2}}{2\sqrt{\pi}}$

$= 1 + \frac{\sqrt{2}}{2\sqrt{\pi}} = 1 + \frac{1}{\sqrt{2\pi}}$

55. (c) : Let $x = a\sin^3 t$ and $y = a\cos^3 t$

$\frac{dx}{dt} = 3a\sin^2 t\cos t$ and $\frac{dy}{dt} = -3a\cos^2 t\sin t$

$\therefore \quad \frac{dy}{dx} = \frac{-3a\cos^2 t\sin t}{3a\sin^2 t\cos t} = -\cot t$

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{d}{dx}(-\cot t) = \frac{d}{dt}(-\cot t)\times\frac{dt}{dx}$

$= \text{cosec}^2 t \cdot \frac{1}{3a\sin^2 t\cos t}$

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{1}{3a}\frac{1}{\sin^4 t\cos t}$

$\therefore \quad \left[\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}} = \frac{1}{3a}(\sqrt{2})^4 \times (\sqrt{2}) = \frac{(2^2)\sqrt{2}}{3a} = \frac{4\sqrt{2}}{3a}$

56. (a) : $y = \sin(2\sin^{-1}x)$

$\frac{dy}{dx} = y_1 = 2\cos(2\sin^{-1}x)\times\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \quad (1 - x^2)y_1^2 = 4(1 - \sin^2(2\sin^{-1}x)) = 4 - 4y^2$

$\Rightarrow \quad (1 - x^2)2y_1y_2 + y_1^2(-2x) = -8yy_1$

$\Rightarrow \quad (1 - x^2)y_2 - xy_1 + 4y = 0$

57. (b) : $x^2 + y^2 = 1 \quad \Rightarrow \quad 2x + 2yy' = 0$

$\Rightarrow \quad 2 + 2(yy'' + y'^2) = 0$

$\Rightarrow \quad yy'' + (y')^2 + 1 = 0$

58. (b) : We have, $\lim\limits_{x\to 1}\frac{f(x)-2}{x^2-1} = \pi$

$\Rightarrow \quad \lim\limits_{x\to 1}[f(x) - 2] = \pi\lim\limits_{x\to 1}(x^2 - 1) = 0$

$\Rightarrow \quad \lim\limits_{x\to 1} f(x) = 2$

59. (c) : We have, $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots. + \frac{x^2}{2} + x + 1$

$\Rightarrow \quad f'(x) = x^{99} + x^{98} + \ldots. + x + 1$

$\therefore \quad f'(0) = 1$

60. (d): $\lim_{x\to 2}\dfrac{f(4)-f(x^2)}{x-2}\quad\left[\dfrac{0}{0}\right]$

$=\lim_{x\to 2}-2x\,f'(x^2)=-2\lim_{x\to 2}x\,f'(x^2)$

$=-2\times 2f'(4)=-2\times 2\times 5=-20$

61. (d): Given that $x=\dfrac{1}{3}t^3-\dfrac{1}{2}\cos 2t+3\cdot 5$

When $t = 0$, then $x=-\dfrac{1}{2}+3\cdot 5=3$

When $t = \pi/2$, then $x=\dfrac{\pi^3}{24}-\dfrac{1}{2}(-1)+3\cdot 5$

$=\dfrac{\pi^3}{24}+\dfrac{1}{2}+\dfrac{7}{2}=\dfrac{\pi^3}{24}+4$

$\therefore$ The distance travelled by the particle between $t = 0$ and $t = \pi/2$ is $\dfrac{\pi^3}{24}+4-3=\dfrac{\pi^3}{24}+1$

62. (d): $f(x) = \sin x - bx + c$

$f'(x) = \cos x - b$

Since $f(x)$ is decreasing then $f'(x) < 0$

i.e. $\cos x - b < 0$ *i.e.* $b > \cos x$

i.e. $b > 1$ [$\because$ the maximum value of $\cos x = 1$]

63. (c): Let the line $ax + by + c = 0$ ... (i)

touches the curve $xy = 4$

i.e. $xy = 2^2$ ... (ii) at the point $\left(2t, \dfrac{2}{t}\right)$

$\left[\because \text{any point on a rectangular hyperbola } xy=c^2 \text{is}\left(ct, \dfrac{c}{t}\right)\right]$

Now differentiating (ii), we get

$x\dfrac{dy}{dx}+y=0 \Rightarrow \dfrac{dy}{dx}=-\dfrac{y}{x}$

Now, $\left(\dfrac{dy}{dx}\right)_{\left(2t,\frac{2}{t}\right)}=\dfrac{-\frac{2}{t}}{2t}=-\dfrac{1}{t^2}$

$\therefore\quad -\dfrac{a}{b}=-\dfrac{1}{t^2}\quad\left[\because \text{slope of (i) is } \dfrac{-a}{b}\right]$

$\Rightarrow t^2=\dfrac{b}{a}\quad \because t^2>0\quad \therefore \dfrac{b}{a}>0$

$\therefore$ Both a and b are of same sign.

i.e. either $a > 0, b > 0$ or $a < 0, b < 0$.

64. (b): $f(x)=\int_1^x t(t-3)(2t-3)^2\,dt$

$\therefore\quad f'(x) = x(x - 3)(2x - 3)^2$

$f'(x) = 0$ gives $x = 0, 3, 3/2$

$f''(x) = (x - 3)(2x - 3)^2 + x(2x - 3)^2 + 2x(x - 3)(2x - 3)2$

$\therefore\quad f''(0) = -27 < 0$

$f''(3) = 3 \times 9 = 27 > 0 \quad\Rightarrow\quad f''\left(\dfrac{3}{2}\right)=0$

Thus the function will be minimum at $x = 3$.

65. (c): $\int\dfrac{x+1}{x(xe^x+1)^2}dx=\int\dfrac{e^x(x+1)dx}{xe^x(1+xe^x)^2}$, Put $xe^x = z$

$=\int\dfrac{dz}{z(1+z)^2}$

Let $\dfrac{1}{z(1+z)^2}=\dfrac{A}{z}+\dfrac{B}{1+z}+\dfrac{C}{(1+z)^2}$

$\Rightarrow\quad 1 = A(1 + z)^2 + Bz(1 + z) + Cz$

Putting $z = 0$, we get $A = 1$

Putting $z = -1$, we get $C = -1$

Equating the co-efficient of z^2, we get

$A + B = 0 \Rightarrow B = -A = -1$

$\therefore\quad \int\dfrac{dz}{z(1+z)^2}=\int\dfrac{dz}{z}-\int\dfrac{dz}{1+z}-\int\dfrac{dz}{(1+z)^2}$

$=\log\left|\dfrac{z}{1+z}\right|+\dfrac{1}{1+z}$

$=\log\left|\dfrac{xe^x}{1+xe^x}\right|+\dfrac{1}{1+xe^x}+c$

66. (d): $\int\left\{\dfrac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx$ Putting $\log_e x = z$

$=\int\dfrac{(z-1)^2}{(1+z^2)^2}\cdot e^z dz=\int e^z\left[\dfrac{1+z^2-2z}{(1+z^2)^2}\right]dz$

$=\int e^z\left[\dfrac{1}{1+z^2}-\dfrac{2z}{(1+z^2)^2}\right]dz$

$=\dfrac{e^z}{1+z^2}+c=\dfrac{x}{1+(\log_e x)^2}+c$

67. (a): $\int_0^{t^2} x\,f(x)dx=\dfrac{2}{5}t^5$

Now differentiating both sides, we get

$t^2 f(t^2)\cdot 2t=\dfrac{2}{5}\cdot 5t^4 \qquad \therefore \qquad f(t^2) = t$

$\Rightarrow\quad f\left(\dfrac{4}{25}\right)=f\left\{\left(\dfrac{2}{5}\right)^2\right\}=\dfrac{2}{5}$

68. (d): $\int_2^4\log[x]dx=\int_2^3\log 2dx+\int_3^4\log 3dx$

$=\log 2\left[x\right]_2^3+\log 3\left[x\right]_3^4$

$=\log 2 + \log 3 = \log 6$

69. (b) : $\int_{\pi/6}^{\pi/3} f(x)dx = \int_{\pi/6}^{\pi/4} \cos x dx + \int_{\pi/4}^{\pi/3} \sin x dx$

$[\because f(x) = \max\{\sin x, \cos x\}]$

$= [\sin x]_{\pi/6}^{\pi/4} + [-\cos x]_{\pi/4}^{\pi/3}$

$= \left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right) + \left(-\frac{1}{2} + \frac{1}{\sqrt{2}}\right) = 2\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right) = \sqrt{2} - 1$

70. (a) : Let the centre of the circle be $(\alpha, 0)$.

$\therefore$ Its equation is

$(x - \alpha)^2 + y^2 = \alpha^2 \Rightarrow x + y\frac{dy}{dx} = \alpha$ ... (i)

Also,

$(x - \alpha)^2 + y^2 = \alpha^2$ gives $x^2 - 2\alpha x + y^2 = 0$ ... (ii)

From (i) and (ii), we get

$y^2 = x^2 + 2xy\frac{dy}{dx}$

71. (a, b, c, d) : Here,

$1 + \sec\theta = \frac{1 + \cos\theta}{\cos\theta} = \frac{2\cos^2(\theta/2)}{\cos\theta}$

$1 + \sec 2\theta = \frac{1 + \cos 2\theta}{\cos 2\theta} = \frac{2\cos^2\theta}{\cos 2\theta}$

Similarly for others,

Now, $\tan\frac{\theta}{2}(1 + \sec\theta)(1 + \sec 2\theta)(1 + \sec 4\theta)....(1 + \sec 2^n\theta)$

$= \tan\frac{\theta}{2} \cdot \frac{2\cos^2(\theta/2)}{\cos\theta} \cdot \frac{2\cos^2\theta}{\cos 2\theta} .... \frac{2\cos^2 2^{n-1}\theta}{\cos 2^n\theta}$

$= \tan\frac{\theta}{2} \cdot 2^{n+1}(\cos\theta \cdot \cos 2\theta ... \cos 2^{n-1}\theta) \cdot \frac{\cos^2(\theta/2)}{\cos 2^n\theta}$

$= \frac{\sin\theta}{\cos 2^n\theta} \cdot 2^n \left\{\frac{2\sin\theta\cos\theta \cdot \cos 2\theta .... \cos 2^{n-1}\theta}{2\sin\theta}\right\}$

$= .....$

$= \frac{\sin\theta}{\cos 2^n\theta} \cdot 2^n \cdot \frac{\sin 2^n\theta}{2^n \sin\theta} = \tan 2^n\theta$

$\therefore$ $f_n(\theta) = \tan 2^n\theta$

So, $f_2\left(\frac{\pi}{16}\right) = \tan\left(4 \cdot \frac{\pi}{16}\right) = 1$

$f_3\left(\frac{\pi}{32}\right) = \tan\left(8 \cdot \frac{\pi}{32}\right) = 1$ and so on.

72. (b) : $5x - y$, $2x + y$ and $x + 2y \in$ A.P.

$\therefore$ $2(2x + y) = 5x - y + x + 2y$

$\Rightarrow$ $2x = y$ ...(i)

Again, $(x - y)^2$, $(xy + 1)$, $(y + 1)^2 \in$ G.P.

$\Rightarrow$ $(xy + 1)^2 = (x - y)^2(y + 1)^2$ ...(ii)

Putting $y = 2x$ in (ii), we get $(x - 1)(4x^2 + x + 1) = 0$

$\therefore$ $x = 1$. $\because$ $4x^2 + x + 1 = 0$ is not satisfied by any $x \in R$.

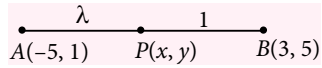
$\therefore$ When $x = 1$, then from (i), we get $y = 2$.

So, $x + y = 1 + 2 = 3$

73. (a, c) : Let P be (x, y)

$\therefore$ $x = \frac{3\lambda - 5}{\lambda + 1}$

and $y = \frac{5\lambda + 1}{\lambda + 1}$



Now, area of the triangle PQR is

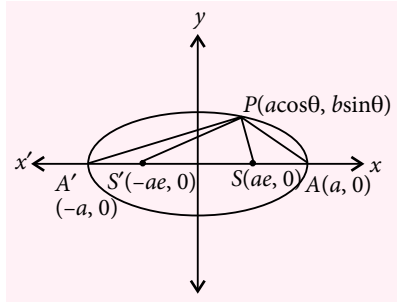
$\frac{1}{2}\left\{\frac{3\lambda - 5}{\lambda + 1}(5 - 2) + 1\left(2 - \frac{5\lambda + 1}{\lambda + 1}\right) + 7\left(\frac{5\lambda + 1}{\lambda + 1} - 5\right)\right\}$

$= \frac{3\lambda - 21}{\lambda + 1}$ [on simplification]

By the problem, $\left|\frac{3\lambda - 21}{\lambda + 1}\right| = 2$

$\Rightarrow$ $\lambda = 23$ and $\lambda = \frac{19}{5}$

74. (b, c) : Let $P(a\cos\theta, b\sin\theta)$ be a point on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



$\therefore$ $\frac{ar(\Delta AA'P)}{ar(\Delta SS'P)} = \frac{ab\sin\theta}{abe\sin\theta} = \frac{1}{e}$

- $K > 1$ as $e < 1$
- K is only dependent on e.

75. (a, b, c, d) : Consider the function

$F(x) = e^{\alpha x} f(x)$

$F(0) = F(1) = 0$ $[\because f(0) = f(1) = 0]$

Hence by Rolle's theorem, $F'(x)$ vanishes for c in $(0, 1)$.

But $F'(x) = e^{\alpha x} \cdot f'(x) + \alpha e^{\alpha x} f(x)$

$= e^{\alpha x}(f'(x) + \alpha f(x))$

$\therefore$ $e^{\alpha c}(f'(c) + \alpha f(c)) = 0$

$\Rightarrow$ $f'(c) + \alpha f(c) = 0$, $c \in (0, 1)$

and α is arbitrary.

76. (c, d) : Let $I = \int\sqrt{1 + \sin 2x}\, dx$

$= \int\sqrt{(\sin x + \cos x)^2} = \int|\sin x + \cos x|\, dx$

Now, $\sin x+\cos x=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)>0,\ x\in\left[-\frac{\pi}{4},\frac{3\pi}{4}\right]$

$\therefore\quad I=\int(\sin x+\cos x)dx=\sin x-\cos x+c$

Further $\sin x+\cos x=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)<0,\ x\in\left[\frac{3\pi}{4},\frac{7\pi}{4}\right]$

$\therefore\quad I=\int-(\sin x+\cos x)dx=\cos x-\sin x+c$

77. (b, d) : $f(x)=\int_{-1}^{x}t(e^t-1)(t-1)(t-2)^3(t-3)^5dt$

$\therefore\quad f'(x)=x(e^x-1)(x-1)(x-2)^3(x-3)^5$

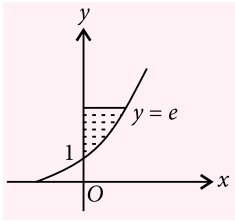
$f(x)\to\infty$ as $x\to\infty$, hence $x=3$, minimum ;

$x=2$, maximum ; $x=1$, minimum

$$x(e^x-1)=x^2\left(1+\frac{x}{2}+....\right)$$

$\therefore\quad x=0$ is an inflectional point.

78. (b, c, d) :



$$A=\int_1^e x\,dy=\int_1^e \ln y\,dy$$

$$=\int_1^e \ln(e+1-y)dy\quad\left[\because\int_a^b f(y)dy=\int_a^b f(a+b-y)dy\right]$$

Further, $A=e\times1-\int_0^1 e^x dx=e-\int_0^1 e^x dx$

79. (b, c) : $(\vec{a}\times\vec{b})^2=(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{b})=\begin{vmatrix}\vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b}\\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b}\end{vmatrix}$

$$=\begin{vmatrix}16 & 4\cdot2\cdot\frac{\sqrt{3}}{2}\\ 4\cdot2\cdot\frac{\sqrt{3}}{2} & 4\end{vmatrix}=64-48=16=|\vec{a}|^2$$

80. (a, c) : Given that $P(A\cup B)\ge\frac{3}{4}$ ... (i)

and $\frac{1}{8}\le P(A\cap B)\le\frac{3}{8}$ ... (ii)

From (i), we get $\frac{3}{4}\le P(A\cup B)\le1$

$\Rightarrow\quad\frac{3}{4}\le P(A)+P(B)-P(A\cap B)\le1$

As the minimum value of $P(A\cap B)=1/8$

$\therefore\quad P(A)+P(B)-\frac{1}{8}\ge\frac{3}{4}$

$\Rightarrow\quad P(A)+P(B)\ge\frac{1}{8}+\frac{3}{4}=\frac{7}{8}$

Again, the maximum value of $P(A\cap B)=3/8$, we get

$$P(A)+P(B)-\frac{3}{8}\le1$$

$\Rightarrow\quad P(A)+P(B)\le1+\frac{3}{8}=\frac{11}{8}$

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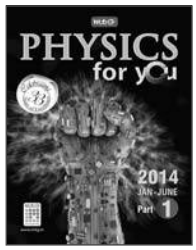

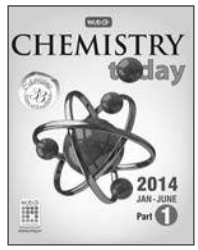

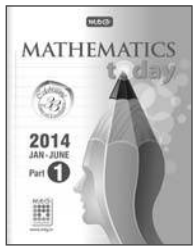

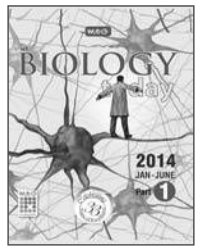

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ISI

Exam on : 10th May

PRACTICE PAPER

Indian Statistical Institute

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PART A

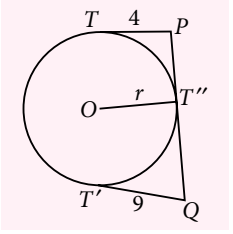
1. A circle of radius r is inscribed in a right isosceles triangle, and a circle of radius R is circumscribed about the triangle. Then R/r equals

(a) $1+\sqrt{2}$ (b) $\dfrac{2+\sqrt{2}}{2}$

(c) $\dfrac{\sqrt{2}-1}{2}$ (d) $\dfrac{1+\sqrt{2}}{2}$

(e) $2(2-\sqrt{2})$

2. In the adjoining figure TP and $T'Q$ are parallel tangents to a circle of radius r, with T and T' the points of tangency. $PT''Q$ is a third tangent with T'' as point of tangency. If $TP = 4$ and $T'Q = 9$, then r is

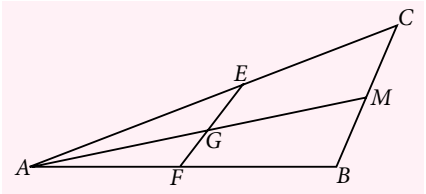


(a) 25/6 (b) 6 (c) 25/4

(d) a number other than 25/6, 6, 25/4

(e) not determinable from the given information

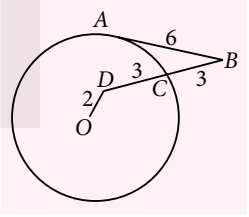
3. In triangle ABC shown in the adjoining figure, M is the midpoint of side BC, $AB = 12$ and $AC = 16$. Points E and F are taken on AC and AB, respectively, and lines EF and AM intersect at G. If $AE = 2AF$, then $\dfrac{EG}{GF}$ equals



(a) 3/2 (b) 4/3 (c) 5/4 (d) 6/5

(e) Not enough information given to solve the problem

4. In the adjoining figure, AB is tangent at A to the circle with center O; point D is interior to the circle; and DB intersects the circle at C. If $BC = DC = 3$, $OD = 2$ and $AB = 6$, then the radius of the circle is



(a) $3+\sqrt{3}$ (b) $\dfrac{15}{\pi}$ (c) $\dfrac{9}{2}$ (d) $2\sqrt{6}$

(e) $\sqrt{22}$

5. Let a, b, c and d be the lengths of sides MN, NP, PQ and QM, respectively, of quadrilateral $MNPQ$. If A is the area of $MNPQ$, then

(a) $A=\left(\dfrac{a+c}{2}\right)\left(\dfrac{b+d}{2}\right)$ if and only if $MNPQ$ is a convex

(b) $A=\left(\dfrac{a+c}{2}\right)\left(\dfrac{b+d}{2}\right)$ if and only if $MNPQ$ is a rectangle

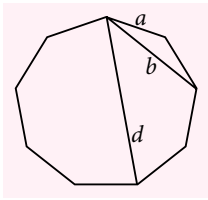
(c) $A\le\left(\dfrac{a+c}{2}\right)\left(\dfrac{b+d}{2}\right)$ if and only if $MNPQ$ is a rectangle

(d) $A\le\left(\dfrac{a+c}{2}\right)\left(\dfrac{b+d}{2}\right)$ if and only if $MNPQ$ is a parallelogram

(e) $A\ge\left(\dfrac{a+c}{2}\right)\left(\dfrac{b+d}{2}\right)$ if and only if $MNPQ$ is a parallelogram

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91). He trains IIT and Olympiad aspirants.

6. If a, b and d are the lengths of a side, a shortest diagonal and a longest diagonal respectively, of a regular nonagon (see adjoining figure), then



(a) $d = a + b$ (b) $d^2 = a^2 + b^2$

(c) $d^2 = a^2 + ab + b^2$ (d) $b = \dfrac{a+d}{2}$

(e) $b^2 = ad$

7. Sides AB, BC, CD and DA respectively, of convex quadrilateral $ABCD$ are extended past B, C, D and A to points B', C', D' and A'. Also, $AB = BB' = 6$, $BC = CC' = 7$, $CD = DD' = 8$ and $DA = AA' = 9$; and the area of $ABCD$ is 10. The area of $A'B'C'D'$ is

(a) 20 (b) 40 (c) 45 (d) 50

(e) 60

8. The function f satisfies the functional equation

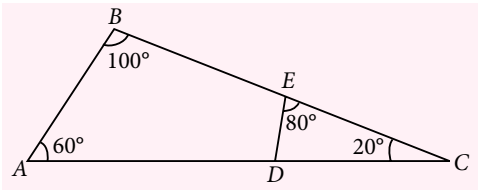
$$f(x) + f(y) = f(x + y) - xy - 1$$

for every pair x, y of real number. If $f(1) = 1$, then the number of integers $n \neq 1$ for which $f(n) = n$ is

(a) 0 (b) 1 (c) 2 (d) 3

(e) Infinite

9. In ΔABC, E is the midpoint of side BC and D is on side AC. If the length of AC is 1 and $\angle BAC = 60°$, $\angle ABC = 100°$, $\angle ACB = 20°$ and $\angle DEC = 80°$, then the area of ΔABC plus twice the area of ΔCDE equals



(a) $\dfrac{1}{4}\cos 10°$ (b) $\dfrac{\sqrt{3}}{8}$

(c) $\dfrac{1}{4}\cos 40°$ (d) $\dfrac{1}{4}\cos 50°$

(e) $\dfrac{1}{8}$

10. If $b > 1$, $\sin x > 0$, $\cos x > 0$ and $\log_b \sin x = a$, then $\log_b \cos x$ equals

(a) $2\log_b\left(1 - b^{\frac{a}{2}}\right)$ (b) $\sqrt{1-a^2}$

(c) b^{a^2} (d) $\dfrac{1}{2}\log_b(1 - b^{2a})$

(e) None of these

11. In the non-decreasing sequence of odd integers $\{a_1, a_2, a_3, ...\} = \{1, 3, 3, 3, 5, 5, 5, 5, 5, ...\}$ each positive odd integer k appears k times. It is a fact that there are integers b, c and d such that, for all positive integers n, $a_n = b[\sqrt{n+c}] + d$, where $[x]$ denotes the largest integer not exceeding x. The sum $b + c + d$ equals

(a) 0 (b) 1 (c) 2 (d) 3

(e) 4

12. How many ordered triples (x, y, z) of integers satisfy the system of equations below?

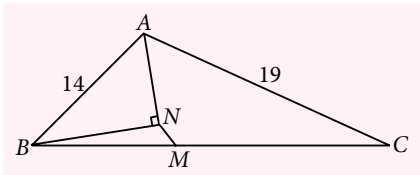
$$x^2 - 3xy + 2y^2 - z^2 = 31,$$
$$-x^2 + 6yz + 2z^2 = 44, x^2 + xy + 8z^2 = 100.$$

(a) 0 (b) 1 (c) 2

(d) A finite number greater than two

(e) Infinitely many

13. In ΔABC, M is the midpoint of side BC, AN bisects $\angle BAC$, $BN \perp AN$ and θ is the measure of $\angle BAC$. If sides AB and AC have lengths 14 and 19 respectively, then length MN equals



(a) 2 (b) 5/2

(c) $\dfrac{5}{2} - \sin\theta$ (d) $\dfrac{5}{2} - \dfrac{1}{2}\sin\theta$

(e) $\dfrac{5}{2} - \dfrac{1}{2}\sin\left(\dfrac{\theta}{2}\right)$

14. Alice, Bob and Carol repeatedly take turns tossing a die. Alice begins; Bob always follows Alice; Carol always follows Bob; and Alice always follows Carol. Find the probability that Carol will be the first one to toss a six. (The probability of obtaining a six on any toss is 1/6, independent of the outcome of any other toss.)

(a) 1/3 (b) 2/9 (c) 5/18 (d) 25/91

(e) 36/91

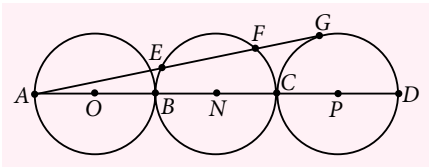
15. Find the units digit in the decimal expansion of

$$(15 + \sqrt{220})^{19} + (15 + \sqrt{220})^{82}$$

(a) 0 (b) 2 (c) 5 (d) 9

(e) None of these

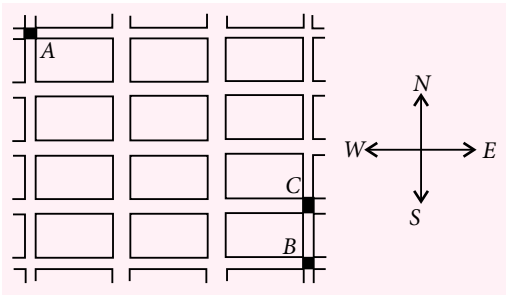
16. In the adjoining figure, points B and C lie on line segment AD, and AB, BC and CD are diameters of circles O, N and P, respectively. Circles O, N and P all have radius 15, and the line AG is tangent to circle P at G. If AG intersects circle N at points E and F, then chord EF has length



(a) 20 (b) $15\sqrt{2}$ (c) 24 (d) 25

(e) None of these

17. The adjoining figure is a map of part of a city: the small rectangles are blocks and the spaces in between are streets. Each morning a student walks from intersection A to intersection B, always walking along streets shown, always going east or south. For variety, at each intersection where he has a choice, he chooses with probability 1/2 (independent of all other choices) whether to go east or south. Find the probability that, on any given morning, he walks through intersection C.



(a) $\frac{11}{32}$ (b) $\frac{1}{2}$ (c) $\frac{4}{7}$ (d) $\frac{21}{32}$

(e) $\frac{3}{4}$

18. Suppose $z = a + ib$ is a solution of the polynomial equation

$$c_4z^4 + ic_3z^3 + c_2z^2 + ic_1z + c_0 = 0$$

where c_0, c_1, c_2, c_3, c_4, a and b are real constants and $i^2 = -1$. Which one of the following must also be a solution ?

(a) $-a - ib$ (b) $a - ib$

(c) $-a + ib$ (d) $b + ia$

(e) None of these

PART B

1. Show that if n is congruent to 2 or 3 modulo 4, then is it not possible to get a rearrangement $(x_1, x_2, ..., x_n)$ of $(1, 2, ..., n)$ such that all are distinct?

2. Let $p(x) = x^2 + ax + b$ be a quadratic polynomial in which a and b are integers. Given any integer n, show that there is an integer m such that $p(n)p(n + 1) = p(m)$.

3. Let AC and BD be two chords of a circle with center O such that they intersect at right-angles inside the circle at the point M. Suppose K and L are the mid-points of the chords AB and CD respectively. Prove that $OKML$ is a parallelogram.

4. Find a finite sequence of 16 numbers such that

(a) it reads the same from left to right as from right to left,

(b) the sum of any 7 consecutive terms is –1 and

(c) the sum of any 11 consecutive terms is + 1.

5. I have six friends and during a certain vacation, I met them during several dinners. I found that I dined with all the six exactly on one day; with every five of them on 2 days; with every four on 3 days; with every three on 4 days : with every two on 5 days. Further every friend was present at 7 dinners and every friend was absent at 7 dinners. How many dinners did I have alone?

6. In an examination 70% students of a class failed in Tamil, 75% in English, 80% in Mathematics and 85% in Science. What is the least percentage of the students who failed in all four subjects?

7. Determine with proof, all the positive integers n for which

(i) n is not the square of any integer and

(ii) $[\sqrt{n}]^3$ divides n^2.

($[x]$ denotes the largest integer that is less than or equal to x)

8. Solve for integers x, y, z:

$x + y = 1 - z$, $x^3 + y^3 = 1 - z^2$.

9. Two circles C_1 and C_2 intersect at two distinct points P and Q in a plane. Let a line passing through P meet the circles C_1 and C_2 in A and B respectively. Let Y be the midpoint of AB and QY meet the circles C_1 and C_2 in X and Z respectively. Show that Y is also the midpoint of XZ.

10. A triangle ABC has incentre I. Points X, Y are located on the line segments AB, AC respectively so that $BX \cdot AB = IB^2$ and $CY \cdot AC = IC^2$. Given that X, I, Y are collinear, find the possible values of the measure of angle A.

11. Let A be a subset of $\{1, 2, 3, ..., 2n-1, 2n\}$ containing $n+1$ elements. Show that

(i) Some two elements of A are relatively prime

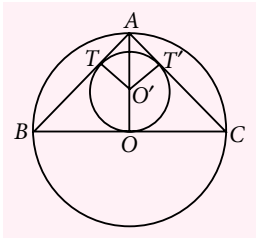
(ii) Some two elements of A have the property that one divides the other.

12. In a class of 25 students, there are 17 cyclists, 13 swimmers and 8 weight lifters and no one is all the three. In a certain mathematics examination 6 students got grades D or E. If the cyclists, swimmers and weight lifters all got grade B or C, determine the number of students who got grade A. Also, find the number of cyclists who are swimmers.

SOLUTION

PART A

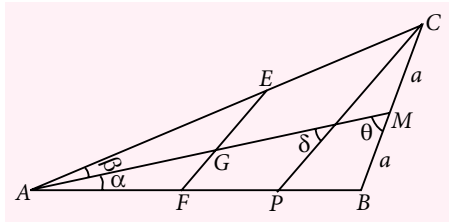
1. (a) : In the adjoining figure, ΔABC is a right isosceles triangle, with $\angle BAC = 90°$ and $AB = AC$, inscribed in a circle with center O and radius R. The line segment AO has length R and bisects line segment BC and $\angle BAC$. A circle with center O' lying on AO and radius r is inscribed in ΔABC. The sides AB and AC are tangent to the inscribed circle with points of tangency T and T', respectively. Since $\Delta ATO'$ has angles 45° – 45° – 90° and $O'T = r$, we have $AT = r$ and $O'A = R - r = r\sqrt{2}$. Hence $R = r + r\sqrt{2}$ and $\frac{R}{r} = 1 + \sqrt{2}$



2. (b) : Since $TP = T''P$, $OT = OT'' = r$, and $\angle PTO = \angle PT''O = 90°$, we have $\Delta OTP \cong \Delta OT''P$. Similarly $\Delta OT'Q \cong \Delta OT''Q$. Letting $x = \angle TOP = \angle POT''$ and $y = \angle T''OQ = \angle QOT'$, we obtain $2x + 2y = 180°$. But this implies that $\angle POQ = x + y = 90°$. Therefore ΔPOQ is a right triangle with altitude OT''. Since the altitude drawn to the hypotenuse of a right triangle is the mean proportion of the segments it cuts, we have

$$\frac{4}{r} = \frac{r}{9} \quad \text{or} \quad r = 6$$

3. (a) : 1st Solution: Construct line CP parallel to EF and intersecting AB at P. Then $\frac{AP}{AC} = \frac{AF}{AE}$, that is, $\frac{AP}{16} = \frac{AF}{2AF}$, so $AP = 8$.



Let a, x, y, α, β, δ and θ be as shown in the adjoining diagram. The desired ratio $\frac{EG}{GF}$ is the same as $\frac{y}{x}$ which we now determine. By the law of sines,

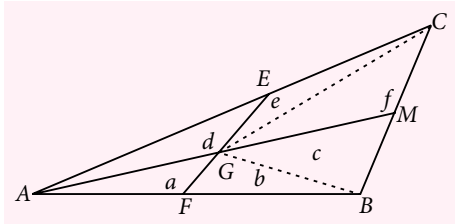
$$\frac{a}{\sin\alpha} = \frac{12}{\sin\theta}; \quad \frac{a}{\sin\beta} = \frac{16}{\sin(180° - \theta)} = \frac{16}{\sin\theta}$$

Hence, $\frac{\sin\beta}{\sin\alpha} = \frac{3}{4}$.

Moreover,

$$\frac{x}{\sin\alpha} = \frac{8}{\sin\delta}; \quad \frac{y}{\sin\beta} = \frac{16}{\sin(180° - \delta)} = \frac{16}{\sin\delta}$$

Hence, $\frac{y}{x} = 2\frac{\sin\beta}{\sin\alpha} = \frac{3}{2}$



OR

Join GB and GC. Triangle ABC is subdivided into six smaller triangles whose areas are denoted by a, b, c, d, e, f, as indicated in the diagram. Triangles AEG and AFG have the common vertex A, so their areas are in the ratio EG to GF. Thus $\frac{EG}{GF} = \frac{d}{a}$, and this we now calculate.

Triangles ACM and ABM have equal areas, so

$$d + e + f = a + b + c$$

Similarly $f = c$, and hence $d + e = a + b$ by subtraction. Let x be the length AF, so that $AE = 2x$, $FB = 12 - x$, $EC = 16 - 2x$. Then

$$\frac{b}{a} = \frac{FB}{FA} = \frac{12 - x}{x} \quad \text{and} \quad \frac{e}{d} = \frac{EC}{EA} = \frac{16 - 2x}{2x}$$

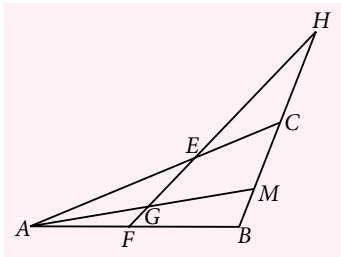
Hence

$$a+b=a+\frac{12-x}{x}a=a\frac{12}{x}$$

and $d+e=d+\frac{16-2x}{2x}d=d\frac{16}{2x}$.

Thus $a + b = d + e$ becomes

$$a\frac{12}{x}=d\frac{16}{2x} \text{ or } 3a=2d, \text{ so } \frac{d}{a}=\frac{3}{2}$$



2nd Solution: Extend *BC* and *FE* until they intersect in a point *H*; see the adjoining figure. The collinear points *A*, *G*, *M* lie on sides (or their extensions) *BF*, *FH*, *HB* of ΔFBH respectively. They also lie on extensions of sides *CE*, *EH*, *HC* of ΔECH. We may therefore apply Menelaus's theorem and find

$$\frac{HG}{FG}\cdot\frac{FA}{BA}\cdot\frac{BM}{HM}=1,\ \frac{HG}{EG}\cdot\frac{EA}{CA}\cdot\frac{CM}{HM}=1$$

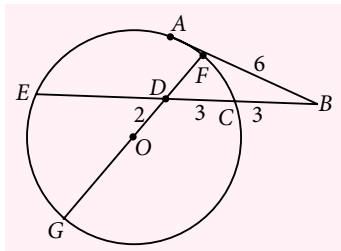
Since *CM* = *BM* and *EA* = 2*FA*, division of the first equation by the second yields

$$\frac{EG}{FG}=2\frac{BA}{CA}=2.\frac{12}{16}=\frac{3}{2}$$

4. (e) : In the adjoining figure, *E* is the point of intersection of the circle and the extension of *DB*, and *FG* is the diameter passing through *D*. Let *r* denote the radius of the circle.

Then, $(BC)(BE) = (AB)^2$

$3(DE + 6) = 36,\ DE = 6.$



Also $(DE)(DC) = (DF)(DG)$,

$6\cdot3 = (r - 2)(r + 2)$,

$18 = r^2 - 4,\ r=\sqrt{22}$.

5. (b) : If *MNPQ* is convex, then *A* is the sum of areas of the triangles into which *MNPQ* is divided by diagonal *MP*, so that

$$A=\frac{1}{2}ab\sin N+\frac{1}{2}cd\sin Q\ .$$

Similarly, dividing *MNPQ* with diagonal *NQ* yields

$$A=\frac{1}{2}ad\sin M+\frac{1}{2}bc\sin P$$

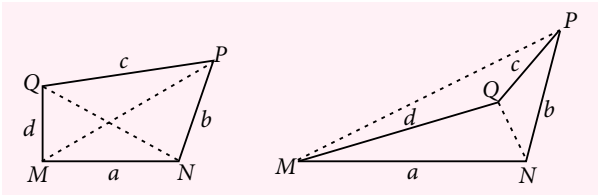
We show below that these two equations for *A* hold also if *MNPQ* is not convex. Therefore, in any case,

$$A\le\frac{1}{4}(ab+cd+ad+bc)=\frac{a+c}{2}.\frac{b+d}{2}$$

The inequality is an equality if and only if

$\sin M = \sin N = \sin P = \sin Q = 1$,

i.e. if and only if *MNPQ* is a rectangle.



If *MNPQ* is not convex, for example if interior angle *Q* of quadrilateral *MNPQ* is greater than 180° as shown in Figure 2, then *A* is the difference

Area of ΔMNP – area of ΔMQP

so that $A=\frac{1}{2}ab\sin N-\frac{1}{2}cd\sin\angle PQM$

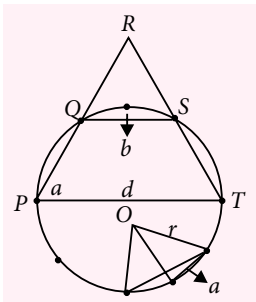
$$=\frac{1}{2}ab\sin N-\frac{1}{2}cd\sin(360^\circ-\angle MQP)$$

$$=\frac{1}{2}ab\sin N+\frac{1}{2}cd\sin\angle MQP$$

6. (a) : 1st Solution: In the adjoining figure, sides *PQ* and *TS* of the regular nonagon have extended to meet at *R* and the circumscribed circle has been drawn. Each side of the nonagon subtends an arc of $\frac{360^\circ}{9}=40^\circ$; therefore $\angle TPQ = \angle STP = \frac{1}{2}\cdot3\cdot40^\circ=60^\circ$.

Since *QS* || *PT*, it follows that both ΔPRT and ΔQRS are equilateral. Hence

$d = PT = PR = PQ + QR = PQ + QS = a + b.$



2nd Solution: Inscribe the nonagon in a circle of radius r. Since the chords of lengths a, b, d subtend central angles of 40°, 80°, 160° respectively, we have

$a = 2r\sin20°$, $b = 2r\sin40°$, $d = 2r\sin80°$.

By means of the identity

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

with $x = 40°$, $y = 20°$

we obtain $\sin40° + \sin20° = 2\sin30°\cos10°$.

Since $\sin30° = 1/2$, this yields

$\sin40° + \sin20° = \cos10° = \sin80°$,

and therefore $b + a = d$.

7. (d): Since the length of base AA' of $\Delta AA'B$ is the same as the length of base AD of ΔABD, and the corresponding altitude of $\Delta AA'B'$ has twice the length of the corresponding altitude of ΔABD.

Area $\Delta AA'B' = 2$area ΔADB,

(Alternatively, we could let θ be the measure of $\angle DAB$ and odserve

$$\text{Area } \Delta AA'B' = \frac{1}{2}(AD)(2AB)\sin(180°-\theta)$$

$$= 2\left[\frac{1}{2}(AD)(AB)\sin\theta\right] = 2 \text{ area } \Delta ABD)$$

Similarly, area $\Delta BB'C' = 2$ area ΔBAC,

area $\Delta CC'D' = 2$ area ΔCBD,

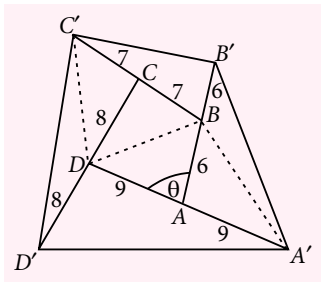
area $\Delta DD'A' = 2$ area ΔDCA.

Therefore, area $A'B'C'D'$

$=$(area $\Delta AA'B'$ + area $\Delta BB'C'$) + (area $\Delta CC'D'$ + area $\Delta DD'A'$) + area $ABCD$

$= 2$(area ΔABD + area ΔBAC) + 2(area ΔCBD + area ΔDCA) + area $ABCD$

$= 5$ area $ABCD = 50$.



8. (b): 1st Solution: Substitute $x = 1$ into the functional equation and solve for the first term on the right side to obtain

$f(y + 1) = f(y) + y + 2$.

Since $f(1) = 1$, one sees by successively substituting $y = 2, 3, 4, \ldots$ that $f(y) > 0$ for every positive integer. Therefore, for y a positive integer,

$f(y + 1) > y + 2 > y + 1$,

and $f(n) = n$ has no solutions for integers $n > 1$. Solving the above equation for $f(y)$ yields

$f(y) = f(y + 1) - (y + 2)$

Successively substituting $y = 0, -1, -2, \ldots$ into this equation yields $f(0) = -1$, $f(-1) = -2$, $f(-2) = -2$, $f(-3) = -1$, $f(-4) = 1$. Now $f(-4) > 0$ and, for $y < -4$, $-(y + 2) > 0$. Thus, for $y < -4$, $f(y) > 0$.

Therefore, $f(n) \neq n$ for $n < -4$; and the solutions $n = 1, -2$ are the only ones.

2nd Solution: We write the functional equation in the form

$f(x + y) - f(y) = f(x) + xy + 1$.

Setting $x = 1$ and using the given value $f(1) = 1$, we find

$f(y + 1) - f(y) = y + 2$

We now set $y = 0, 1, 2, \ldots, n$ successively, then $y = 0, -1, -2, \ldots, -m$ successively, and obtain

$f(1) - f(0) = 2$ (so $f(0) = -1$)	$f(0) - f(-1) = 1$
$f(2) - f(1) = 3$	$f(-1) - f(-2) = 0$
.....	.....
$f(n) - f(n-1) = n + 1$	$f(-(m-1)) - f(-m)$ $= -(m - 2)$.

Adding the set of equations in the left column and cancelling like terms with opposite signs yields

$$f(n) - f(0) = 2 + 3 + \ldots + n + 1 = -1 + \sum_{i=1}^{n+1} i$$

Recalling that the sum of the first k positive integers is $\frac{1}{2}k(k+1)$ and using the value $f(0) = -1$, we obtain

$$f(n) + 1 = \frac{(n+1)(n+2)}{2} - 1$$

(1). $f(n) = \frac{1}{2}(n^2 + 3n - 2)$ for each non-negative integer n.

The same procedure applied to the column of equations on the right above shows that equation (1) is valid also for negative integers; thus (1) holds for all integers.

The equation $f(n) = n$ is equivalent to

$n^2 + 3n - 2 = 2n$,

$n^2 + n - 2 = (n + 2)(n - 1) = 0$,

$n = 1$ or $n = -2$;

So there is only one integer solution other than $n = 1$.

Remark: The computations involving the equations in the right column above can be avoided by setting $y = -x$ in the original equation to obtain

$f(x) + f(-x) = f(0) + x^2 - 1$,

or since $f(0) = -1$,

(2). $f(x) = -f(-x) + x^2 - 2$ for all x.

Let x be negative and substitute (1) in (2) to get

$$f(x)=-\frac{1}{2}\left[(-x)^2+3(-x)-2\right]+x^2-2=\frac{1}{2}(x^2+3x-2)$$

Thus negative integers as well as positive satisfy (1). In fact, it is easy to check that the function

$$f(x)=\frac{x^2+3x-2}{2}$$

satisfies the given functional equation for all real x and y.

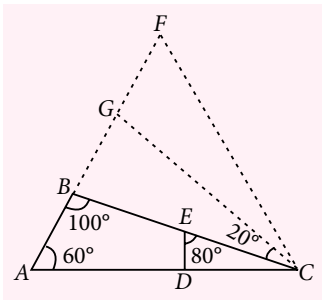
Remark: The technique used in the second solution is frequently applied to solve many commonly occurring functional equations involving the expression $\Delta f(x) = f(x + 1) - f(x)$; $\Delta f(x)$ is called the "first difference" of $f(x)$ and behaves, in many ways, like the first derivative $f'(x)$ of $f(x)$. For example

$$f'(x)=x+2 \text{ implies } f(x)=\frac{x^2}{2}+2x+c$$

$$\Delta f(x)=x+2 \text{ implies } f(x)=\frac{x(x-1)}{2}+2x+c$$

where c is a constant. The study of functions by means of $\Delta f(x)$ is called the theory of finite differences.

9. (b) : Let F be the point on the extension of side AB past B for which $AF = 1$. Since $AF = AC$ and $\angle FAC = 60°$, ΔACF is equilateral. Let G be the point on line segment BF for which $\angle BCG = 20°$. Then ΔBCG is similar to ΔDCE and $BC = 2(EC)$. Also ΔFGC is congruent to ΔABC.



Therefore, area ΔACF = (area ΔABC + area ΔGCF) + area ΔBCG,

$$\frac{\sqrt{3}}{4}=2 \text{ area } (\Delta ABC)+4 \text{ area } (\Delta CDE)$$

$$\frac{\sqrt{3}}{8}=\text{area } (\Delta ABC)+2 \text{ area } (\Delta CDE)$$

10. (d) : $\log_b \sin x = a$; $\sin x = b^a$; $\sin^2 x = b^{2a}$;

$\cos x = (1-b^{2a})^{1/2}$; $\log_b \cos x = \frac{1}{2}\log_b(1-b^{2a})$

11. (c) : The given information implies that $a_n > a_{n-1}$ if and only if $n + c$ is a perfect square. Since $a_2 > a_1$ and $a_5 > a_4$, it follows that $2 + c$ and $5 + c$ are both squares. The only squares differing by 3 are 1 and 4; here $2 + c = 1$, so $c = -1$.

Now $a_2 = 3 = b[\sqrt{2+c}] + d = b[\sqrt{1}] + d = b + d$.

Hence, $b + c + d = 3 - 1 = 2$. (Although we only need to find $b + d$ here, it is easy to see by setting $n = 1$ that $d = 1$, and hence $b = 2$.)

Remark: The last member of the k^{th} string of equal terms occupies the position $1 + 3 + 5 + ... + (2k - 1) = k^2$ in the sequence. Its successor is $a_{k^2+1} = a_{k^2} + 2$.

Therefore, $[\sqrt{k^2+1+c}] - [\sqrt{k^2+c}] = \frac{2}{b} = 1$

so $c = -1$ and $b = 2$.

12. (a) : 1st Solution: If the last equation is multipled by 3 and added to the first equation, we obtain

$4x^2 + 2y^2 + 23z^2 = 331$.

Clearly, z^2 is odd and less than 25, so $z^2 = 1$ or 9. This leads to the two equations

$2x^2 + y^2 = 154$ and $2x^2 + y^2 = 62$

Both of which are quickly found to have no solutions. Note that we made no use here of the second of the original equations.

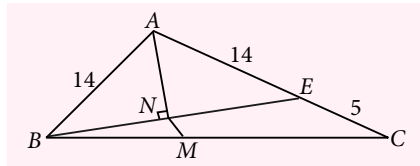
2nd Solution: Adding the given equations and rearranging the terms of the resulting equation yields

$(x^2 - 2xy + y^2) + (y^2 + 6yz + 9z^2) = 175$

or $(x - y)^2 + (y + 3z)^2 = 175$

The square of an even integer is divisible by 4; the square of an odd integer, $(2n + 1)^2$, has remainder 1 when divided by 4. So the sum of two perfect squares can only have 0, 1 or 2 as a remainder when divided by 4. But 175 has remainder 3 upon division by 4, and hence the left and right sides of the equation above cannot be equal. Thus there are no integral solutions.

13. (b) : In the adjoining figure, BN is extended past N and meets AC at E. Triangle BNA is congruent to ΔENA. Since $\angle BAN = \angle EAN$, $AN = AN$ and $\angle ANB = \angle ANE$. Therefore N is the midpoint of BE, and $AB = AE = 14$. Thus $EC = 5$. Since MN is the line joining the midpoints of sides BC and BE of ΔCBE, its length is $\frac{1}{2}(EC) = \frac{5}{2}$.



14. (d) : The probability that the first 6 is tossed on the k^{th} toss is the product

$$\begin{pmatrix}\text{probability that never a 6 was}\\ \text{tossed in the previous } (k\text{-}1) \text{ tosses}\end{pmatrix}\begin{pmatrix}\text{probability that a 6 is}\\ \text{tossed on the } k^{th} \text{ toss}\end{pmatrix}$$

$$= \left(\frac{5}{6}\right)^{k-1}\left(\frac{1}{6}\right).$$

The probability that Carol will toss the first 6 is the sum of the probabilities that she will toss the first 6 on her first turn (3rd toss of the game), on her second turn (6th toss of the game), on her third turn, etc. This sum is

$$\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^5\cdot\frac{1}{6}+\ldots+\left(\frac{5}{6}\right)^{3n-1}\cdot\frac{1}{6}+\ldots,$$

an infinite geometric series with first term $a=\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}$ and common ratio $r=\left(\frac{5}{6}\right)^3$. This sum is

$$\frac{a}{1-r}=\frac{\frac{5^2}{6^3}}{1-\left(\frac{5^3}{6^3}\right)}=\frac{5^2}{6^3-5^3}=\frac{25}{91}$$

15. (d) : Let $d_1=a+\sqrt{b}$ and $d_2=a-\sqrt{b}$, where $a = 15$ and $b = 220$. Then using the binomial theorem, we may obtain

$$d_1^n+d_2^n=2\left[a^n+\binom{n}{2}a^{n-2}b+\binom{n}{4}a^{n-4}b^2+\ldots\right],$$

where n is any positive integer. Since fractional powers of b have been eliminated in this way, and since a and b are both divisible by 5, we may conclude that $d_1^n+d_2^n$ is divisible by 10.

We now apply the above result twice, taking $n = 19$ and $n = 82$. In this way we obtain

$$d_1^{19}+d_2^{19}=10k_1 \text{ and } d_1^{82}+d_2^{82}=10k_2,$$

k_1 and k_2 are positive integers. Adding and rearranging these results gives

$$d_1^{19}+d_2^{19}=10k_1 \text{ and } d_1^{82}+d_2^{82}=10k_2,$$

where $k=k_1+k_2$. But $d_2=15-\sqrt{220}=\frac{5}{15+\sqrt{220}}<\frac{1}{3}$.

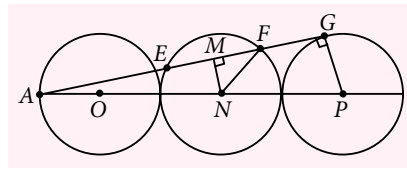
Therefore, $d_2^{19}+d_2^{82}<1$. It follows that the units digit of $10k-(d_2^{19}+d_2^{82})$ is 9.

16. (c) : In the adjoining figure, MN is perpendicular to AG at M, and NF and PG are radii.

Since $\Delta AMN \sim \Delta AGP$, it follows that

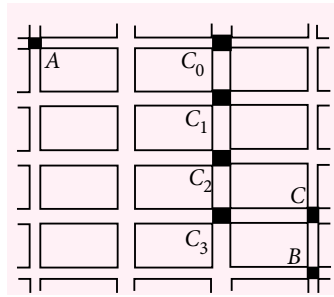
$$\frac{MN}{AN}=\frac{GP}{AP} \quad \text{or,} \quad \frac{MN}{45}=\frac{15}{75}$$

Thus $MN = 9$. Applying the Pythagorean theorem to triangle MNF yields $(MF)^2 = (15)^2 - 9^2 = 144$, so $MF = 12$. Therefore $EF = 24$.



17. (d) : 1st Solution: The probability that the student passes through C is the sum from $i = 0$ to 3 of the probabilities that he enters intersection C_i in the adjoining figure and goes east. The number of paths from A to C_i is $\binom{2+i}{2}$, because each such path has 2 eastward block segments and they can occur in any order. The probability of taking any one of these paths to C_i and then going east is $\left(\frac{1}{2}\right)^{3+i}$ because there are $3 + i$ intersections along the way (including A and C_i) where an independent choice with probability 1/2 is made. So the answer is

$$\sum_{i=0}^{3}\binom{2+i}{2}\left(\frac{1}{2}\right)^{3+i}=\frac{1}{8}+\frac{3}{16}+\frac{6}{32}+\frac{10}{64}=\frac{21}{32}.$$



2nd Solution: One may construct a tree-diagram of the respective probabilities, obtaining the values step-by-step as shown in the scheme to the right (the final 1 also serves as a check on the computations).

$$\begin{array}{cccc}
1 & \to & \frac{1}{2} & \to & \frac{1}{4} & \to & \frac{1}{8} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
\frac{1}{2} & \to & \frac{1}{2} & \to & \frac{3}{8} & \to & \frac{5}{16} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
\frac{1}{4} & \to & \frac{3}{8} & \to & \frac{3}{8} & \to & \frac{1}{2} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
\frac{1}{8} & \to & \frac{1}{4} & \to & \frac{5}{16} & \to & \frac{21}{32} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
\frac{1}{16} & \to & \frac{3}{16} & \to & \frac{11}{32} & \to & 1
\end{array}$$

It is important to recognize that not all twenty of the thirty-five paths leading from A to B through C are equally likely; hence answer (c) is incorrect.

18. (c) : We recall the theorem that complex roots of polynomials with real coefficients come in conjugate pairs. Though not applicable to the given polynomial, that theorem is proved by a technique which we can use to work this problem too. Namely, conjugate both sides of the original equation

$$0 = c_4z^4 + ic_3z^3 + c_2z^2 + ic_1z + c_0\,,$$

obtaining

$$0 = c_4\bar{z}^4 - ic_3\bar{z}^3 + c_2\bar{z}^2 - ic_1\bar{z} + c_0$$

$$= c_4(-\bar{z})^4 + ic_3(-\bar{z})^3 + c_2(-\bar{z})^2 - ic_1(-\bar{z}) + c_0$$

That is, $-\bar{z} = -a+ib$ is also a solution of the original equation. (One may check by example that neither $-a - bi$ nor $a - bi$ nor $b + ai$ need be a solution.) For instance, consider the equation $z^4 - iz^3 = 0$ and the solution $z = i$. Here $a = 0$, $b = 1$. Neither $-i$ nor 1 is a solution. [Alternatively, the substitution $z = i\omega$ into the given equation makes the coefficients real and the above quoted theorem applicable.

PART B

1. Suppose $(x_1, x_2, ..., x_n)$ is a rearrangement of $(1, 2, ..., n)$ such that $|x_1 - 1|, |x_2 - 1|, ..., |x_n - 1|$ are all distinct. Let us show that $n \equiv 0$ or $1 \pmod 4$. Now

$$\sum_{i=1}^{n}(x_i - i) = 0 \qquad ...(1)$$

Since $|x_1 - 1|, |x_2 - 2|, ..., |x_n - n|$ are all distinct, they have to be a rearrangement of $0, 1, ..., n - 1$; therefore,

$$\sum_{i=1}^{n}|x_i - i| = \frac{n(n-1)}{2} \qquad ...(2)$$

Since for any $k \in z|k| \equiv k \pmod 2$,

$$\sum_{i=1}^{n}|x_i - i| \equiv \sum_{i=1}^{n}(x_i - i) \pmod 2$$

Hence, it follows from (1) and (2) that $\frac{n(n-1)}{2}$ is even.

Therefore $4|n(n - 1)$; *i.e.*, 4 divides either n or $n - 1$.

2. 1st Solution: Since $p(n)p(n + 1)$ is of fourth degree n with leading coefficient 1, m, if it exists, must be of the form $n^2 + tn + s$

Therefore $p(0)\, p(1) = p(s)$

$\Rightarrow \quad b^2 + ab + b = s^2 + as + b$

$\Rightarrow \quad s^2 - b^2 = a(b - s)$

$\Rightarrow \quad (s - b)(s + b) = a(b - s) \Rightarrow (s - b)(s + a + b) = 0$

Thus we have $s = b$ or $s = -(a + b)$. Taking $s = b$, we have,

$$(n^2 + an + b)((n + 1)^2 + a(n + 1) + b)$$
$$= (n^2 + tn + b)^2 + a(n^2 + tn + b) + b$$

Equating the coefficients of n^3, we obtain $t = a + 1$. Taking $m = n^2 + (a + 1)n + b$, it can be routinely verified that $p(n)p(n + 1) = p(m)$ for all $n \in N$.

2nd Solution: Writing $p(x) = (x + \alpha)(x + \beta)$, we have $\alpha + \beta = a$ and $\alpha\beta = b$. Now

$(n + \alpha)(n + 1 + \beta) = n^2 + (\alpha + \beta + 1)n + \alpha\beta + \alpha$

$= n^2 + (a + 1)n + b + \alpha$

Similarly $(n + \beta)(n + 1 + \alpha) = m + \beta$

where $m = n^2 + (a + 1)n + b$. Therefore

$p(n)p(n + 1) = (n + \alpha)(n + \beta)\,(n + 1 + \alpha)(n + 1 + \beta)$

$= (m + \alpha)(m + \beta) = p(m)$.

3. 1st Solution: Choose M as the origin. Let A, B, C, D be $(2a, 0)$, $(0, 2b)$, $(2c, 0)$, $(0, 2d)$ respectively. Then

$AM{\cdot}MC = BM{\cdot}MD \quad \Rightarrow \quad ac = bd$

(in magnitude as well as in sign).

O being the intersection of the perpendicular bisectors of AC and BD, is $(a + c, b + d)$. Since K, L are (a, b), (c, d) respectively, the midpoint of KL is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$ which is also the mid-point of OM. Therefore $OKML$ is a parallelogram.

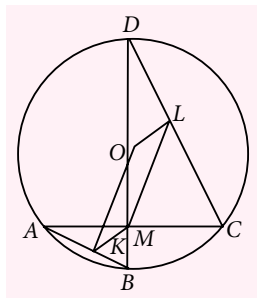
2nd Solution: Let R be the radius of the circle. Then in ΔADB, $AB = 2R\sin\angle ADB$.

Also in ΔCAD, $CD = 2R\sin\angle CAD = 2R\cos\angle ADB$

($\because$ In ΔADM, $\angle ADB + \angle CAD = 90°$). Therefore

$AB^2 + CD^2 = 4R^2$.

Now, $AK^2 + OK^2 = AO^2 \Rightarrow \frac{1}{2}AB^2 + OK^2 = R^2$



and therefore $OK = \frac{1}{2}CD = ML$ ($\because$ In the right angled triangle CMD, L is the mid-point of the hypotenuse).

Similarly $OL = KM$.

3rd Solution: Let $\angle KAM = \theta$.

In ΔAKM, $AK = KM \quad \Rightarrow \quad \angle AMK = \theta$

$\therefore \quad \angle AMK = 180° - 2\theta$.

So. $\angle OKM = 180° - 2\theta - 90° = 90° - 2\theta$

By similar argument, since $\angle MDC = \theta$, we get $\angle OLM = 90° - 2\theta$. Further,

$$\angle KML = \angle KMA + \angle AMD + \angle DML = \theta + 90° + \theta = 90° + 2\theta$$

Thus, three angles $\angle OKM$, $\angle OLM$, $\angle KML$ of $OKML$ are $90° - 2\theta$, $90° - 2\theta$, $90° + 2\theta$ respectively; so the remaining is $90° + 2\theta$ and hence $OKML$ is a parallelogram.

4. Let $(a_i)_{i=1}^{16}$ be one such sequence. For any $k \le 9$, by using (b), we have

$$a_k + a_{k+1} + ... + a_{k+6} = -1 = a_{k+1} + ... + a_{k+7}$$

$\therefore \quad a_k = a_{k+7}$.

Similarly, by using (c), we have $a_k = a_{k+11}$, for all $k \le 5$. Thus by (a), (b) and (c), we have the following data. For all $i, j \le 16$.

If $i + j = 17$ or $|i - j| = 7$ or 11, then $a_i = a_j$.

Using this, we have

$$a_1 = a_8 = a_{15} = a_4 = a_{11} = a_6 = a_{13} = a_2 = a_9 = a_{16} = a_5 = a_{12}$$

and $a_3 = a_{10} = a_7 = a_{14}$

Now using (b) and (c) for the first 7 and 11 terms respectively, we get $5a_1 + 2a_3 = -1$ and $8a_1 + 3a_3 = 1$; solving, we have $a_1 = 5$ and $a_3 = -13$. Thus the sequence which satisfies our requirement is unique, *viz*,

5, 5, – 13, 5, 5, 5, – 13, 5, 5, –13, 5, 5, 5, –13, 5, 5.

5. For $i = 1, 2, ..., 6$. Let A_i(resp. B_i) be the set of days on which i^{th} friend is present (resp. absent) at dinner given that

$$|A_i| = |B_i| = 7,\ |A_i \cap A_j| = 5$$

$$|A_i \cap A_j \cap A_k| = 4,\ \ |A_i \cap A_j \cap A_k \cap A_l| = 3$$

$$|A_i \cap A_j \cap A_k \cap A_l \cap A_m| = 2$$

and $|A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5 \cap A_6| = 1$

Where i, j, k, l, m vary from 1 to 6 and are distinct. Number of dinners at which at least one friend was present

$$= |A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6|$$

$$= \Sigma |A_1| - \Sigma |A_i \cap A_j| + \Sigma |A_i \cap A_j \cap A_k|$$

$$\Sigma |A_i \cap A_j \cap A_k \cap A_l| + \Sigma |A_i \cap A_j \cap A_k \cap A_l \cap A_m| - |A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5 \cap A_6|$$

$$= \binom{6}{1}7 - \binom{6}{2}5 + \binom{6}{3}4 - \binom{6}{4}3 + \binom{6}{5}2 - \binom{6}{6}1 = 13$$

The total number of dinners is $|A_1| + |B_1| = 14$. Hence the number of dinners I had alone is 14 – 13 = 1.

6. Let the strength of the class be 100. Let T, E, M, S be the sets of students who passed in Tamil, English, Mathematics and Science, respectively. Then the number of students who failed in all subjects is

$$100 - |T \cup E \cup M \cup S| \ge 100 - (|T| + |E| + |M| + |S|)$$

$$= 100 - (30 + 25 + 20 + 15) = 10$$

Note that equality holds when

$$|T \cup E \cup M \cup S| = |T| + |E| + |M| + |S|;$$

i.e., when these sets are pairwise disjoint.

7. Let $[\sqrt{n}] = k$. Then $k^2 < n < (k+1)^2$. Also since k^3 divides n^2, we have that k^2 divides n^2 and hence k divides n. Thus the only possibilities for n are $n = k^2 + k$ and $n = k^2 + 2k$.

(i) Let $n = k^2 + k$. Then

$$k^3 | n^2 \ \Rightarrow\ k^3 | (k^2 + k)^2 = k^4 + 2k^3 + k^2$$

$\Rightarrow \quad k^3 | k^2 \ \Rightarrow \ k = 1$ *i.e.*, $n = 2$.

(ii) Let $n = k^2 + 2k$. Then

$$k^3 | n^2 \Rightarrow k^3 | (k^2 + 2k)^2 = k^4 + 4k^3 + 4k^2$$

which implies that $k^3 | 4k^2$ or $k | 4$. Therefore, $k = 1, 2$ or 4. When $k = 1, 2, 4$, we get the corresponding values 3, 8 and 24 for n. Thus $n = 2, 3, 8$ and 24 are all positive integers satisfying the given conditions.

8. Eliminating z from the given set of equations, we get

$$x^3 + y^3 + \{1 - (x + y)\}^2 = 1.$$

This factors to

$$(x + y)(x^2 - xy + y^2 + x + y - 2) = 0$$

Case-I: Suppose $x + y = 0$. Then $z = 1$ and $(x, y, z) = (m, -m, 1)$, where m is an integer give one family of solutions.

Case-II : Suppose $x + y \ne 0$. Then we must have

$$x^2 - xy + y^2 + x + y - 2 = 0.$$

This can be written in the form

$$(2x - y + 1)^2 + 3(y + 1)^2 = 12$$

Here there are two possibilities:

$$2x - y + 1 = 0,\ y + 1 = \pm 2;$$

or $\quad 2x - y + 1 = \pm 3,\ y + 1 = \pm 1$

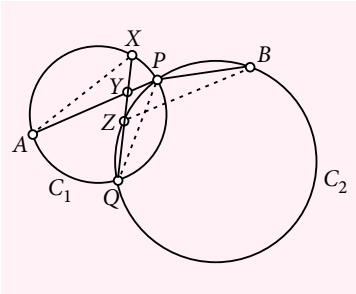
Analysing all these cases, we get the following solutions:

(0, 1, 0), (–2, –3, 6), (1, 0, 0),
(0, –2, 3), (–2, 0, 3), (–3, –2, 6).

9. **1st Solution:** Join PQ, BZ and AX.

In circle C_2, we have $\angle ZBP = \angle ZQP$; and in circle C_1, we have $\angle PQX = \angle PAX$. Thus, we obtain $\angle ZBA = \angle BAX$. (So BZ is parallel to AX.) The triangles AXY and BZY are then congruent, because by hypothesis $AY = YB$ and angles AYX and YAX are respectively equal to BYZ

and YBZ. This congruence gives us $XY = ZY$, which is what we want.



2nd Solution: We use a standard property of intersecting chords of a circle. If AB and CD are two chords of a circle intersecting at O either internally or externally, then $AO \cdot OB = CO \cdot OD$.

In the figure, AP and XQ are two chords of the circle C_1, intersecting externally at Y. So

$$AY \cdot YP = XY \cdot YQ \qquad \ldots. (1)$$

Similarly, BP and QZ are two chords of the circle C_2 intersecting externally at Y. So

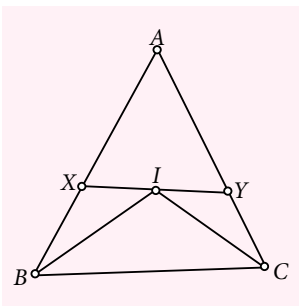
$$YP \cdot YB = YZ \cdot YQ. \qquad \ldots. (2)$$

The left hand side expressions of (1) and (2) are equal because it is given that $AY = YB$. Therefore the right-hand expressions are equal. This give, on cancelling the factor YQ, the desired relation $XY = YZ$.

10. From the relation $BI^2 = BX \cdot BA$, we see that BI is a tangent to the circle passing through A, X, I at I. Hence

$$\angle BIX = \angle BAI = \frac{A}{2} \qquad \ldots. (1)$$

[Alternatively, one observes that in triangles BIX and BAI, $\angle IBX$ is common and $\frac{BI}{BX} = \frac{BA}{BI}$. Consequently the two triangles are similar, implying (1).]



Similarly, from the relation $CI^2 = CY \cdot CA$, we obtain

$$\angle CIY = \angle CAI = \frac{A}{2} \qquad \ldots. (2)$$

It is known that

$$\angle BIC = 90° + \frac{A}{2} \qquad \ldots. (3)$$

From (1), (2), (3) and the fact that X, I, Y are collinear, we obtain

$$\frac{A}{2} + \frac{A}{2} + \left(90° + \frac{A}{2}\right) = 180°.$$

Solving, we get $A = 60°$.

11. (i) Since A contains $(n + 1)$ elements of the set $\{1, 2, 3, ..., 2n\}$ some two of the $n + 1$ element must be consecutive. But then any two consecutive integers are relatively prime and we have the desired conclusion.

(ii) We give a proof by making use of the pigeon-hole principle. Write each of the $n + 1$ numbers in the form $2^p \cdot q$, where q is an odd number and p a non negative integer. What are the possible values of q ? Since the numbers of A come from the set $\{1, 2, 3, \ldots 2n\}$, we see that q can be any one of the n odd numbers 1, 3, 5, 7, ... $(2n - 1)$. As there are $n + 1$ numbers in A, there are $n + 1$ values of q. Hence by the afore-mentioned principle, for some two numbers $a = 2^{p1} \cdot q_1$ and $b = 2^{p2} \cdot q_2$, we must have $q_1 = q_2$. Since $a \neq b$, p_1 is either greater than p_2 or less than p_2. In the former case b divides a, while in the latter case a divides b.

Remark : Strangely, this problem can be solved by induction also.

12. Let S denote the set of all the 25 students in the class, X the set of swimmers in S, Y the set of all weight-lifters and Z the set of all cyclists. Since students in $X \cup Y \cup Z$ all get grades B and C and six students get grades D or E, the number of students in $X \cup Y \cup Z \leq 25 - 6 = 19$. Now assign one point to each of the 17 cyclists, 13 swimmers and 8 weight-lifters. Thus a total of 38 points would be assigned among the students in $X \cup Y \cup Z$. Note that no student can have $|X \cup Y \cup Z| \geq 19$ as otherwise 38 points cannot be accounted for. (For example if there were only 18 student in $(X \cup Y \cup Z)$ the maximum number of points that could be assigned to them is 36). Therefore $|X \cup Y \cup Z| = 19$ and each students $X \cup Y \cup Z$ is in exactly 2 of the sets X, Y, Z. Hence the number of student getting grade $A = 25 - 19 - 6 = 0$, *i.e.* no student gets A grade. Since there are $19 - 8 = 11$ students who are not weight-lifters all these 11 students must be both swimmers and cyclists. (Similarly there are 2 who are both swimmers and weight-lifters and 6 who are both cyclists and weight-lifters).

EXAM ON 14th to 29th MAY

FULL LENGTH

BITSAT

PRACTICE PAPER

PHYSICS

1. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is d. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $K_1 = 3$ and thickness $d/3$ while the other one has dielectric constant $K_2 = 6$ and thickness $2d/3$. Now the capacitance becomes equal to

(a) 20.25 pF (b) 1.8 pF
(c) 45 pF (d) 40.5 pF

2. A large hollow metallic sphere A(of radius R) is positively charged to a potential of 100 V and a small sphere B (of radius $R/5$) is also positively charge to a potential of 100 V. Now B is placed inside A and they are connected by a wire. The final potential of A will be

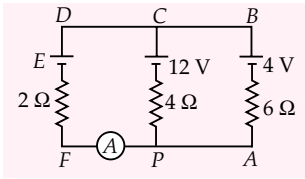
(a) 200 V (b) 150 V
(c) 120 V (d) none of these

3. A motor cycle starts from rest and accelerates along a straight path at 2 m s^{-2}. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest?
(Speed of sound = 330 m s^{-1}).

(a) 49 m (b) 98 m (c) 147 m (d) 196 m

4. $ABCDFPA$ is a network of three batteries of the emfs E, 12 V and 4 V respectively and three resistances 2 Ω, 4 Ω and 6 Ω connected as shown in the figure. An ideal ammeter connected between F and P shows a current reading of 0.5 A. Then the value of the emf E is

(a) 6 V
(b) 6.6 V
(c) 8 V
(d) 5.5 V



5. A 60 cm metal rod M_1 is joined to another 100 cm metal rod M_2 to form an L shaped single piece. This piece is hung on a peg at the joint. The two rods are observed to be equally inclined to the vertical. If the two rods are equally thick, the ratio of density of M_1 to that M_2 is

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $\frac{25}{9}$ (d) $\frac{9}{25}$

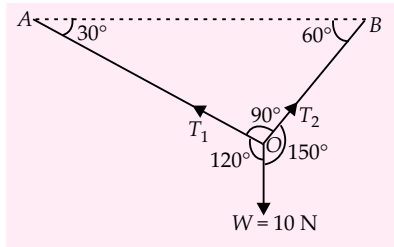
6. The path difference between two interfering waves at a point on the screen is $\lambda/8$. The ratio of the intensity at this point and that at the central fringe will be

(a) 0.853 (b) 8.53 (c) 85.3 (d) 85.

7. A 100 eV electron is fired directly towards a large metal plate having surface charge density -2×10^{-6} C m^{-2}. The distance from where the electrons be projected so that it just fails to strike the plate is

(a) 0.22 mm (b) 0.44 mm
(c) 0.66 mm (d) 0.88 mm

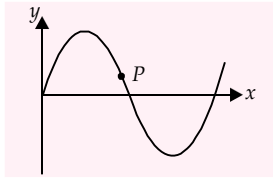
8. A ball of mass 1 kg hangs in equilibrium from two strings OA and OB as shown in figure. What are the tensions in strings OA and OB?
(Take g = 10 m s^{-2})



(a) 5 N, zero (b) zero, $5\sqrt{3}$ N
(c) 5 N, $5\sqrt{3}$ N (d) $5\sqrt{3}$ N, 5 N

9. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm s^{-1}. The wavelength of the wave is

0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of particle P when its displacement is 5 cm is



(a) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m s^{-1} (b) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m s^{-1}

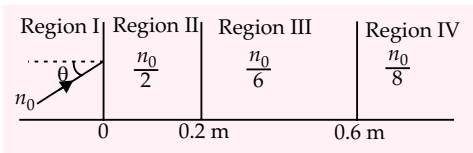
(c) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m s^{-1} (d) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m s^{-1}

10. When a certain metallic surface is illuminated with monochromatic light of wavelength λ, the stopping potential for photoelectric current is $3V_0$ and the stopping potential changes to V_0 if it is illuminated by light of wavelength 2λ. Find the threshold wavelength.

(a) 6λ (b) $\frac{4\lambda}{3}$

(c) 4λ (d) 8λ

11. A light beam is travelling from region I to region IV (Refer figure). The refractive index in regions I, II, III and IV are $n_0, \frac{n_0}{2}, \frac{n_0}{6}$ and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering region IV is



(a) $\sin^{-1}\left(\frac{3}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{8}\right)$

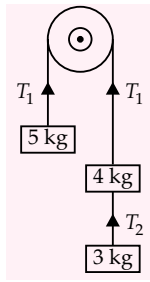
(c) $\sin^{-1}\left(\frac{1}{4}\right)$ (d) $\sin^{-1}\left(\frac{1}{3}\right)$

12. The energy (E), angular momentum (L) and universal gravitational constant (G) are chosen as fundamental quantities. The dimensions of universal gravitational constant in the dimensional formula of Planck's constant (h) is

(a) 0 (b) –1

(c) $\frac{5}{3}$ (d) 1

13. With reference to the figure showing a light inextensible string passing over a fixed frictionless pulley, the tension T_2 is



(a) 24.5 N

(b) 29.4 N

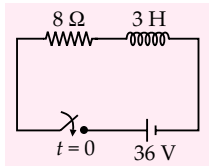
(c) 18.6 N

(d) 68.6 N

14. In a galvanometer 5% of the total current in the circuit passes through it. If the resistance of the galvanometer is G, the shunt resistance S connected to the galvanometer is

(a) 19G (b) $\frac{G}{19}$ (c) 20G (d) $\frac{G}{20}$

15. For the circuit shown in figure, the ratio of the voltage across the resistor to across the inductor when the current in the circuit is 2 A, is



(a) 0.4 (b) 1.6 (c) 0.8 (d) 1.8

16. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of light used) is I. If I_0 denotes the maximum intensity, $\frac{I}{I_0}$ is equal to

(a) $\frac{3}{4}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

17. Two bars A and B of circular cross-section and of same volume and made of the same material are subjected to tension. If the diameter of A is half that of B and if the force applied to both the rods is the same and it is in the elastic limit, the ratio of extension of A to that of B will be

(a) 16 (b) 8 (c) 4 (d) 2

18. A convex lens of focal length 0.15 m is made of a material of refractive index 3/2. When it is placed in a liquid, its focal length is increased by 0.225 m. The refractive index of the liquid is

(a) $\frac{7}{4}$ (b) $\frac{5}{4}$ (c) $\frac{9}{4}$ (d) $\frac{3}{2}$

19. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the forces acting on it is

(a) $2\pi mk^2r^2t$ (b) mk^2r^2t

(c) $\frac{mk^4r^2t^5}{3}$ (d) zero

20. The ratio of specific heat of a gas at constant pressure to that at constant volume is γ. The change in internal energy of one mole of gas when volume changes from V to 2 V at constant pressure P is

(a) $\frac{R}{(\gamma-1)}$ (b) PV

(c) $\frac{PV}{(\gamma-1)}$ (d) $\frac{\gamma PV}{\gamma-1}$

21. If the potential energy of a gas molecule is $U=\frac{M}{r^6}-\frac{N}{r^{12}}$, M and N being positive constants, then the potential energy at equilibrium must be

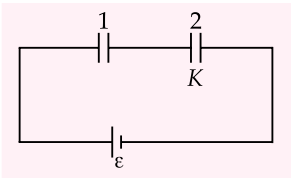
(a) zero (b) $\frac{M^2}{4N}$ (c) $\frac{N^2}{4M}$ (d) $\frac{MN^2}{4}$

22. In Boolean algebra $\overline{(A+\bar{B})\cdot C}$ will be equal to

(a) $(\bar{A}\cdot B)+\bar{C}$ (b) $(A\cdot\bar{B})+C$

(c) $(A\cdot B)\cdot C$ (d) $(A+\bar{B})+C$

23. Two identical capacitors 1 and 2 are connected in series to a battery as shown in figure. Capacitor 2 contains a dielectric slab of constant K. Q_1 and Q_2 are the charges stored in C_1 and C_2. Now, the dielectric slab is removed and the corresponding charges are Q_1' and Q_2'. Then



(a) $\frac{Q_1'}{Q_1}=\frac{K+1}{K}$ (b) $\frac{Q_2'}{Q_2}=\frac{K+1}{2}$

(c) $\frac{Q_2'}{Q_2}=\frac{K+1}{2K}$ (d) $\frac{Q_1'}{Q_1}=\frac{K}{2}$

24. Two satellites S_1 and S_2 are revolving round a planet in coplanar and concentric circular orbits of radii R_1 and R_2 in the same direction respectively. Their respective periods of revolution are 1 hour and 8 hours. The radius of the orbit of satellite S_1 is equal to 10^4 km. Their relative speed when they are closest, in km h^{-1}, is

(a) $\frac{\pi}{2}\times10^4$ (b) $\pi\times10^4$

(c) $2\pi\times10^4$ (d) $4\pi\times10^4$

25. The binding energies of the nuclei of ${}_2^4\text{He}$, ${}_3^7\text{Li}$, ${}_6^{12}\text{C}$ and ${}_7^{14}\text{N}$ are 28, 52, 90 and 98 MeV respectively. Which of these are most stable?

(a) ${}_2^4\text{He}$ (b) ${}_3^7\text{Li}$ (c) ${}_6^{12}\text{C}$ (d) ${}_7^{14}\text{N}$

26. A rotating body has angular momentum L. If its frequency is doubled and kinetic energy is halved, its angular momentum becomes

(a) $2L$ (b) $\frac{L}{2}$

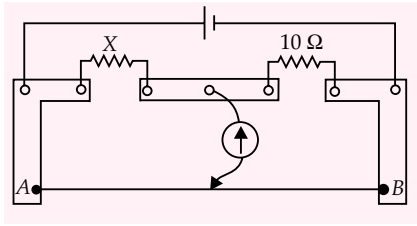
(c) $4L$ (d) $\frac{L}{4}$

27. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is

(a) 1215 Å (b) 1640 Å

(c) 2430 Å (d) 4687 Å

28. A meter bridge is set-up as shown, to determine an unknown resistance X using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of X is



(a) 10.2 ohm (b) 10.6 ohm

(c) 10.8 ohm (d) 11.1 ohm

29. 5.6 litre of helium gas at STP is adiabatically compressed to 0.7 litre. Taking the initial temperature to be T_1, the work done in the process is

(a) $\frac{9}{8}RT_1$ (b) $\frac{3}{2}RT_1$ (c) $\frac{15}{8}RT_1$ (d) $\frac{9}{2}RT_1$

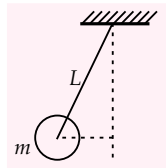
30. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is

(a) 9

(b) 18

(c) 27

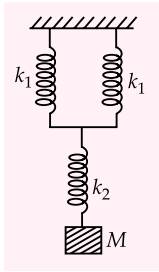
(d) 36



31. A bus is moving with a velocity of 5 m s^{-1} towards a huge wall. The driver sounds a horn of frequency 165 Hz. If the speed of sound in air is 335 m s^{-1}, the number of beats heard per second by a passenger inside the bus will be

(a) 3 (b) 4 (c) 5 (d) 6

32. What will be the spring constant of the spring system shown in the figure?



(a) $\frac{k_1}{2}+k_2$

(b) $\left[\frac{1}{2k_1}+\frac{1}{k_2}\right]^{-1}$

(c) $\frac{1}{2k_1}+\frac{1}{k_2}$

(d) $\left[\frac{2}{k_1}+\frac{1}{k_2}\right]^{-1}$

33. When a charged particle moving with velocity $\vec{v}$ is subjected to a magnetic field $\vec{B}$, the force on it is non-zero. This implies that

(a) angle between $\vec{v}$ and $\vec{B}$ is either zero or 180°.

(b) angle between $\vec{v}$ and $\vec{B}$ is necessarily 90°.

(c) angle between $\vec{v}$ and $\vec{B}$ can have any value other than 90°.

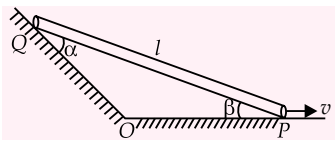
(d) angle between $\vec{v}$ and $\vec{B}$ can have any value other than zero and 180°.

34. The equation of a stationary wave is $y=4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$. The distance between a node and its next antinode is

(a) 7.5 units (b) 1.5 units

(c) 22.5 units (d) 30 units

35. A rod of length l slides down along the inclined wall as shown in figure. At the instant shown in figure, the speed of the end P is v, then the speed of Q will be



(a) $\frac{v\cos\alpha}{\cos\beta}$ (b) $\frac{v\cos\beta}{\cos\alpha}$

(c) $\frac{v\sin\alpha}{\sin\beta}$ (d) $\frac{v\sin\beta}{\sin\alpha}$

36. A plane mirror is placed in y-z plane facing towards negative x-axis. The mirror is moving parallel to y-axis with a speed of 5 cm s^{-1}. A point object P is moving infront of the mirror with a velocity $(3\hat{i}+4\hat{j})$. The velocity of image is

(a) $-3\hat{i}+4\hat{j}$ (b) $3\hat{i}-4\hat{j}$

(c) $-3\hat{i}$ (d) $3\hat{i}+4\hat{j}$

37. A square loop of wire, side length 10 cm is placed at angle of 45° with a magnetic field that changes uniformly from 0.1 T to zero in 0.7 seconds. The induced current in the loop (its resistance is 1 Ω) is

(a) 1.0 mA (b) 2.5 mA

(c) 3.5 mA (d) 4.0 mA

38. The resistance of the wire in the platinum resistance thermometer at ice point is 5 Ω and at steam point is 5.25 Ω. When the thermometer is inserted in an unknown hot bath its resistance is found to be 5.5 Ω. The temperature of the hot bath is

(a) 100°C (b) 200°C

(c) 300°C (d) 350°C

39. A ball is dropped from a bridge at a height of 176.4 m over a river. After 2 s, a second ball is thrown straight downwards. What should be the initial velocity of the second ball so that both hit the water simultaneously?

(a) 2.45 m s^{-1} (b) 49 m s^{-1}

(c) 14.5 m s^{-1} (d) 24.5 m s^{-1}

40. A flywheel rotates with a uniform angular acceleration. Its angular velocity increases from 20π rad s^{-1} to 40π rad s^{-1} in 10 seconds. How many rotations did it make in this period?

(a) 80 (b) 100 (c) 120 (d) 150

CHEMISTRY

41. Equation of Boyle's law is

(a) $\frac{dP}{P}=-\frac{dV}{V}$ (b) $\frac{dP}{P}=+\frac{dV}{V}$

(c) $\frac{d^2P}{P}=-\frac{dV}{dT}$ (d) $\frac{d^2P}{P}=+\frac{d^2V}{dt}$

42. When Br^- ion is reacted with conc. H_2SO_4, then the product formed will be

(a) Br_2 (b) HBr

(c) HBr and Br_2 (d) HBr, Br_2 and SO_2

43. The units of van der Waals constants a and b respectively are

(a) L atm^2 mol^{-1} and mol L^{-1}

(b) L atm $mol^2$and mol L

(c) L^2 atm mol^{-2} and mol^{-1} L

(d) L^{-2} atm^{-1} mol^{-1} and L mol^{-2}

44. The electron gain enthalpies of B, C, N and O with negative sign are in the order
(a) B < C < N < O (b) B < C < O > N
(c) B < C > O > N (d) B > C < O < N

45. A dihalogen derivative *X* of a hydrocarbon with three carbon atoms reacts with alcoholic KOH and produces another hydrocarbon which forms a red precipitate with ammoniacal Cu_2Cl_2. *X* gives an aldehyde on reaction with aqueous KOH. The compound *X* is
(a) 1,3-dichloropropane
(b) 1,2-dichloropropane
(c) 2,2-dichloropropane
(d) 1,1-dichloropropane

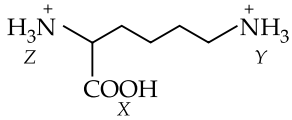
46. Which of the following compounds of phosphorus contain P—O—P bond?
I. Pyrophosphorous acid
II. Hypophosphoric acid
III. Metaphosphoric acid
IV. Orthophosphorous acid
(a) I only (b) I and III only
(c) II and IV only (d) III only

47. The order of reactivity of various alkyl halides towards nucleophilic substitution follows the order
(a) *R*—I > *R*—Br > *R*—Cl > *R*—F
(b) *R*—F > *R*—Cl > *R*—Br > *R*—I
(c) *R*—Cl > *R*—Br > *R*—I > *R*—F
(d) *R*—Br > *R*—I > *R*—Cl > *R*—F

48. A solution which is 10^{-3} M each in Mn^{2+}, Fe^{2+}, Zn^{2+} and Hg^{2+} is treated with 10^{-16} M sulphide ion. If K_{sp} of MnS, FeS, ZnS and HgS are 10^{-13}, 10^{-18}, 10^{-24} and 10^{-53} respectively, which one will precipitate first?
(a) FeS (b) MnS (c) HgS (d) ZnS

49. In PO_4^{3-}, the formal charge on each oxygen atom and the P — O bond order respectively are
(a) – 0.75, 1 (b) – 2, 1.0
(c) – 0.75, 1.25 (d) – 3, 1.25

50. The correct order of decreasing acid strengths of different groups in the given amino acid is



(a) *X* > *Z* > *Y* (b) *Z* > *X* > *Y*
(c) *X* > *Y* > *Z* (d) *Y* > *X* > *Z*

51. The atomic numbers of vanadium (V), chromium (Cr), manganese (Mn) and iron (Fe) are 23, 24, 25 and 26 respectively. Which one of these may be expected to have the highest second ionisation enthalpy?
(a) V (b) Cr (c) Mn (d) Fe

52. The mass of molecule *A* is twice the mass of molecule *B*. The rms speed of *A* is twice the rms speed of *B*. If two samples of *A* and *B* contain same number of molecules, the ratio of pressures of two samples *A* and *B* in separate containers of equal volume is
(a) 8 (b) 4 (c) 16 (d) 2

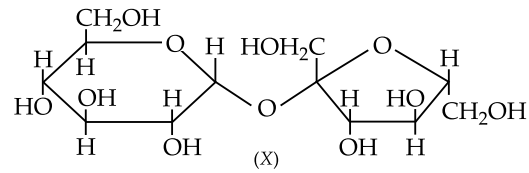
53. For the following reaction occurring in an automobile
$2C_8H_{18(g)} + 25O_{2(g)} \rightarrow 16CO_{2(g)} + 18H_2O_{(g)}$; the sign of ΔH, ΔS and ΔG would be
(a) –, +, + (b) +, +, –
(c) +, –, + (d) –, +, –

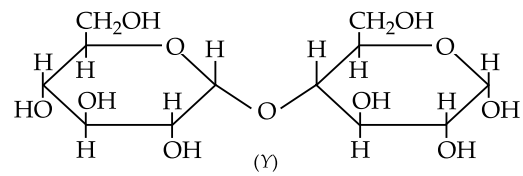
54. Which of the following polymer is stored in the liver of animals?
(a) Amylose (b) Cellulose
(c) Amylopectin (d) Glycogen

55. Which complex is likely to show optical activity?
(a) *trans*-$[Co(NH_3)_4Cl_2]^+$
(b) $[Cr(H_2O)_6]^{3+}$
(c) *cis*-$[Co(NH_3)_2(en)_2]^{3+}$
(d) *trans*-$[Co(NH_3)_2(en)_2]^{3+}$

56. The van der Waals' constants for four gases *P*, *Q*, *R* and *S* are 4.17, 3.59, 6.71 and 3.8 atm L^2 mol^{-2}. Therefore, the ascending order of their liquefaction is
(a) *R* < *P* < *S* < *Q* (b) *Q* < *S* < *R* < *P*
(c) *Q* < *S* < *P* < *R* (d) *R* < *P* < *Q* < *S*

57. Which of the following will not reduce Tollen's reagent?





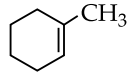
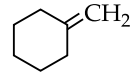
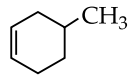
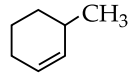
(a) (*X*) only (b) (*Y*) only
(c) Both (*X*) and (*Y*)
(d) Neither (*X*) nor (*Y*)

58. The hybridisation of atomic orbitals of the transition metals in the following complexes are respectively

$[Fe(H_2O)_6]^{3+}$, $[Co(NH_3)_6]^{3+}$, $[Ni(CN)_4]^{2-}$, $[Ni(CO)_4]$

(a) d^2sp^3, sp^3d^2, dsp^2, sp^3
(b) sp^3d^2, d^2sp^3, sp^3, dsp^2
(c) sp^3d^2, d^2sp^3, dsp^2, sp^3
(d) d^2sp^3, sp^3d^2, sp^3, dsp^2

59. A hydrocarbon *P* of the formula C_7H_{12} on ozonolysis gives a compound *Q* which undergoes aldol condensation giving 1-acetylcyclopentene. The compound *P* is

(a) 
(b) 
(c) 
(d) 

60. A dust particle having mass equal to 10^{-11} g, diameter 10^{-4} cm and velocity 10^{-4} cm s^{-1}. If the error in measurement of velocity is 0.1% then the uncertainty in its position is

(a) 5.57×10^{-10} cm (b) 5.27×10^{-6} m
(c) 5.27×10^{-6} cm (d) 5.27×10^{-10} m

61. The solubility of sulphates in water down the IIA group follows the order

Be > Mg > Ca > Sr > Ba.

This is due to

(a) increase in melting point
(b) increasing molecular mass
(c) decreasing lattice energy
(d) high heat of solvation of smaller ions.

62. The correct order of ionic radii of Y^{3+}, La^{3+}, Eu^{3+} and Lu^{3+} is

(a) $Y^{3+} < La^{3+} < Eu^{3+} < Lu^{3+}$
(b) $Y^{3+} < Lu^{3+} < Eu^{3+} < La^{3+}$
(c) $Lu^{3+} < Eu^{3+} < La^{3+} < Y^{3+}$
(d) $La^{3+} < Eu^{3+} < Lu^{3+} < Y^{3+}$

63. The compound that does not give iodoform test is

(a) ethanol (b) ethanal
(c) methanol (d) propanone.

64. A hexapeptide has the composition Ala, Gly, Phe, Val. Both the N-terminal and C-terminal units are Val. Cleavage of the hexapeptide by chemotrypsin gives two different tripeptides, both having Val as the N-terminal group. Among the products of random hydrolysis, one is Ala-Val dipeptide fragment. What is the primary structure of the hexapeptide?

(a) Val-Gly-Phe-Val-Ala-Val
(b) Val-Ala-Phe-Val-Gly-Val
(c) Val-Gly-Ala-Val-Phe-Val
(d) Val-Phe-Val-Ala-Gly-Val

65. Four electrons, identified by quantum numbers *n* and *l* (i) $n = 4$, $l = 1$; (ii) $n = 4$, $l = 0$; (iii) $n = 3$, $l = 2$; (iv) $n = 3$, $l = 1$ can be placed in order of increasing energy, from the lowest to highest, as

(a) (iv) < (ii) < (iii) < (i)
(b) (ii) < (iv) < (i) < (iii)
(c) (i) < (iii) < (ii) < (iv)
(d) (iii) < (i) < (iv) < (ii)

66. Which of the following unit cells is most unsymmetrical?

(a) Triclinic (b) Orthorhombic
(c) Monoclinic (d) Hexagonal

67. Which of the following amides will not undergo Hofmann bromamide reaction?

(a) CH_3CONH_2 (b) $CH_3CH_2CONH_2$
(c) $C_6H_5CONH_2$ (d) $CH_3CONHCH_3$

68. Two weak acid solutions HA_1 and HA_2 each with the same concentration and having pK_a values 3 and 5 are placed in contact with hydrogen electrode (1 atm, 25°C) and are interconnected through a salt bridge. The emf of the cell is

(a) 0.21 V (b) 0.059 V
(c) 0.018 V (d) 0.021 V

69. The maximum number of isomers for an alkane with the molecular formula C_5H_{12} is

(a) 2 (b) 5 (c) 4 (d) 3

70. Which of the following pairs consists of species with same bond order?

(a) C_2, O_2 (b) N_2, O_2
(c) B_2, F_2 (d) Both (a) and (c).

71. The number of *d*-electrons in Fe^{2+} is not equal to that of the

(a) *p*-electrons in Cl^- (b) *s*-electrons in Fe^{3+}
(c) *s*-electrons in Mg (d) *p*-electrons in Ne.

72. Orlon is a polymer of

(a) styrene (b) tetrafluoroethylene
(c) vinyl chloride (d) acrylonitrile.

73. For which of the following compounds Lassaigne's test of nitrogen will fail?
(a) $H_2NCONH.NH_2.HCl$
(b) $H_2NNH_2.2HCl$
(c) H_2NCONH_2
(d) $C_6H_5—N=N—C_6H_5$

74. Which of the following arrangements represent increasing oxidation number of the central atom?
(a) CrO_2^-, ClO_3^-, CrO_4^{2-}, MnO_4^-
(b) ClO_3^-, CrO_4^{2-}, MnO_4^-, CrO_2^-
(c) CrO_2^-, ClO_3^-, MnO_4^-, CrO_4^{2-}
(d) CrO_4^{2-}, MnO_4^-, CrO_2^-, ClO_3^-

75. What is the value of $1/n$, in Freundlich adsorption isotherm?
(a) Between 2 and 4 in all cases
(b) Between 0 and 1 in all cases
(c) 1 in case of chemisorptions
(d) 1 in case of physical adsorption

76. According to IUPAC system, what is the correct name of the compound $[Cr(NH_3)_3(H_2O)_3]Cl_3$?
(a) Triamminetriaquachromium(III) chloride
(b) Triamminetriaquachromium chloride(III)
(c) Tetraammoniumtriaquachromium(III) chloride
(d) None of the above.

77. The relative ease of dehydration of alcohols follows following order
(a) tertiary > secondary > primary
(b) primary > secondary > tertiary
(c) secondary > tertiary > primary
(d) tertiary > primary > secondary.

78. 45 g of ethylene glycol ($C_2H_6O_2$) is mixed with 600 g of water, what is the depression of freezing point?
(a) 7.9 K (b) 2.5 K
(c) 6.6 K (d) 2.2 K

79. The concentration of hydrogen ion in a sample of soft drink is 3.8×10^{-3} M. What is its pH?
(a) 3.84 (b) 2.42
(c) 4.44 (d) 1.42

80. A metal M readily forms water soluble sulphate MSO_4, water insoluble hydroxide $M(OH)_2$ and oxide MO which becomes inert on heating. The hydroxide is soluble in NaOH. The metal M is
(a) Be (b) Mg
(c) Ca (d) Sr

MATHEMATICS

81. If α and β ($\alpha < \beta$), are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then
(a) $0 < \alpha < \beta$ (b) $\alpha < 0 < \beta < |\alpha|$
(c) $\alpha < \beta < 0$ (d) $\alpha < 0 < |\alpha| < \beta$

82. Let z and ω be two complex numbers such that $|z| \le 1$, $|\omega| \le 1$ and $|z - i\bar{\omega}| = |z + i\omega| = 2$ then z equals
(a) 1 or i (b) i or $-i$
(c) 1 or -1 (d) i or -1

83. If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation
(a) $0 < M \le 1$ (b) $1 \le M \le 2$
(c) $2 \le M \le 3$ (d) $3 \le M \le 4$

84. If C_r stands for ${}^nC_r$, then the sum of the series

$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2 - 2C_1^2 + \ldots\ldots\ldots + (-1)^n (n+1) C_n^2\right]$$

where n is an even positive integer, is equal to
(a) 0 (b) $(-1)^{\frac{n}{2}}(n+1)$
(c) $(-1)^n (n + 2)$ (d) none of these

85. In arrangements of the word 'MONDAY', if p = number of words that do not begin with M and q = number of words which begin with M but do not end with Y, then $p : q =$
(a) 25 : 4 (b) 4 : 25
(c) < 0 (d) none of these

86. If $S_n = \sum_{r=1}^{n} 2^{r-1}$, then $\sum_{r=1}^{n} \log_{2^r}(S_r + 1) =$
(a) n (b) $\frac{n}{2}$
(c) $\frac{n(n+1)}{2}$ (d) none of these

87. If $A = \begin{bmatrix} \sqrt[3]{8} & 0 & 0 \\ 0 & \sqrt[3]{2} & 0 \\ 0 & 0 & \frac{1}{\sqrt[3]{4}} \end{bmatrix}$ and $|\text{adj}(\text{adj}A)| = \sqrt[3]{\lambda^{\lambda}}$, then $\lambda =$
(a) 12 (b) 4
(c) 2 (d) none of these

88. If $A=\begin{pmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{pmatrix}$ and I be unit matrix of order 2, then $(I-A)\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}=$

(a) I (b) $I+A$ (c) $A-I$ (d) 0

89. If n_1 and n_2 be randomly chosen from positive integers and probability that $3^{n_1}+3^{n_2}$ and $8^{n_1}+8^{n_2}$ each is divisible by 5 be p_1 and p_2 respectively, then

(a) $p_1 = p_2$ (b) $p_1 < p_2$
(c) $p_1 > p_2$ (d) Nothing can be said

90. If $\frac{a^2+1-\cos^2 x}{\cos 2x}+\frac{a^2}{1-\cot^2 x}=0$ has atleast one real solution of x then a lies in

(a) $(-1, 1)$ (b) $[-1, 1]$
(c) $\{0\}$ (d) none of these

91. If $f(x)=\frac{\sqrt{1+\cos^2 x-3\sin x\cos x}}{\cos x}$ is defined then $x\in$

(a) $\left(n\pi-\frac{\pi}{2}, n\pi+\frac{\pi}{2}\right)$ (b) $\left[n\pi+\frac{\pi}{4}, n\pi+\tan^{-1}2\right]$

(c) $R-\left(n\pi+\frac{\pi}{4}, n\pi+\tan^{-1}2\right)-\left\{n\pi-\frac{\pi}{2}\right\}$

(d) none of these

92. Find the general solution for $|\sin\theta-\cos^2\theta|\geq|\sin^2\theta-3\sin\theta+3|+4|1-\sin\theta|$.

(a) $n\pi$; $n\in I$ (b) $2n\pi$; $n\in I$

(c) $(4n+1)\frac{\pi}{2}; n\in I$ (d) none of these

93. If $f(x)=\begin{cases} \frac{(5^x-1)^3}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x^2}{3}\right)}, & x\neq 0 \\ 9(\log_e 5)^3, & x=0 \end{cases}$ is continuous at $x = 0$ then value of 'a' equals

(a) 3 (b) 2
(c) 1 (d) None of these

94. If $\int f(x)\sin x\cos x\,dx=\frac{1}{2(b^2-a^2)}\log(f(x))+c$, then $\int f(x)\,dx$ equals

(a) $\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)$ (b) $ab\tan^{-1}\left(\frac{a\tan x}{b}\right)$

(c) $\frac{1}{ab}\tan^{-1}\left(\frac{b\tan x}{a}\right)$ (d) $\frac{1}{ab}\tan^{-1}\left(\tan\left(\frac{bx}{a}\right)\right)$

95. If $g(x) = \sin[\pi^3]x + \sin[-\pi^3]x$, $[\cdot]$ denotes the greatest integer function, then $\int_0^{\pi/2}\pi\sin^4 x\cdot g\left(-\frac{\pi}{2}\right)dx$ equals

(a) $\frac{\pi^2}{16}$ (b) $\frac{\pi^2}{8}$ (c) $\frac{3\pi^2}{16}$ (d) $\frac{3\pi^2}{32}$

96. The point of contact of the circles $x^2+y^2-4x+6y-3=0$ and $x^2+y^2+16x+6y+37=0$ is

(a) $(-8, -3)$ (b) $(2, -3)$
(c) $(-2, -3)$ (d) None of these

97. The co-ordinates of the point on the parabola $y = x^2 + 10x + 3$ which is nearest to the straight line $y = 4x - 7$ are

(a) (3, 18) (b) (18, 3)
(c) (−3, −18) (d) (−18, −3)

98. The equation of the plane through the line of intersection of $\vec{r}\cdot(\hat{i}-2\hat{j}+2\hat{k}) = 1$ and $\vec{r}\cdot(2\hat{i}-\hat{j}) + 2 = 0$ and $\perp$ to $\vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k}) + 9 = 0$ is

(a) $\vec{r}\cdot(-5\hat{i}+\hat{j}+2\hat{k}) = 7$
(b) $\vec{r}\cdot(5\hat{i}+\hat{j}+2\hat{k}) = 7$
(c) $\vec{r}\cdot(5\hat{i}-\hat{j}-2\hat{k}) = 7$
(d) None of these

99. Let $\vec{a}=\alpha_1\hat{i}+\alpha_2\hat{j}+\alpha_3\hat{k}$, $\vec{b}=\beta_1\hat{i}+\beta_2\hat{j}+\beta_3\hat{k}$ and $\vec{c}=\gamma_1\hat{i}+\gamma_2\hat{j}+\gamma_3\hat{k}$, $|\vec{a}|=2\sqrt{2}$ makes angle $\frac{\pi}{3}$ with plane of $\vec{b}$ and $\vec{c}$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix}^n$ is equal to (n is even natural number).

(a) $\left(\frac{|\vec{b}||\vec{a}|}{2\times 3}\right)^{n/2}$ (b) $\left(\frac{\sqrt{3}\,|\vec{b}||\vec{c}|}{\sqrt{2}}\right)^n$

(c) $\frac{\left(|\vec{b}||\vec{c}|\right)^{n/2}}{\sqrt{3}\,2^n}$ (d) none of these

100. The mean value of the numbers $\frac{{}^{30}C_0}{1}, \frac{{}^{30}C_2}{3}, \ldots, \frac{{}^{30}C_{20}}{21}, \frac{{}^{30}C_{21}}{22}, \ldots, \frac{{}^{30}C_{30}}{31}$ equals

(a) $\frac{2^{31}}{31}$ (b) $\frac{4^{13}}{31}$
(c) $\frac{2^{13}}{31}$ (d) None of these

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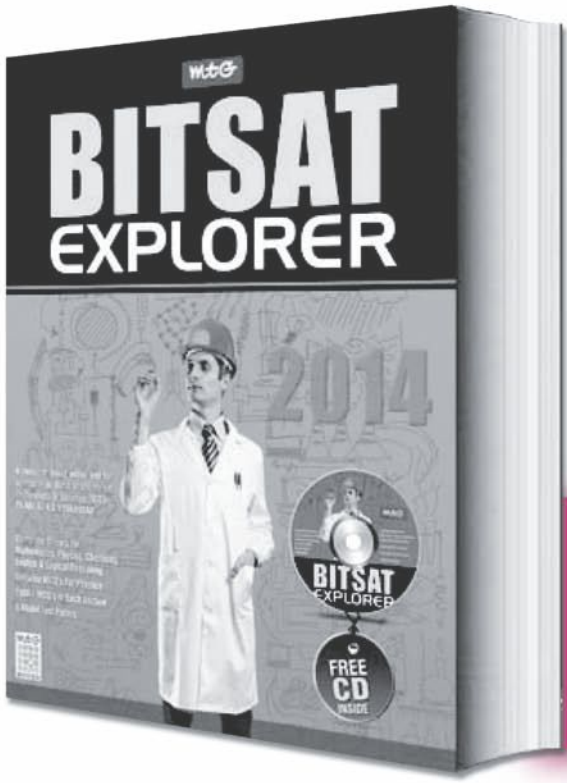

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101. Let S be a non-empty subset of R. Consider the following statement:
P : There is a rational number $x \in S$ such that $x > 0$.
Which of the following statements is the negation of the statement P ?
(a) There is a rational number $x \in S$ such that $x \le 0$.
(b) There is no rational number $x \in S$ such that $x \le 0$.
(c) Every rational number $x \in S$ satisfies $x \le 0$.
(d) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational.

102. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is
(a) $4\sqrt{2} - 2$ (b) $4\sqrt{2} + 2$
(c) $4\sqrt{2} - 1$ (d) $4\sqrt{2} + 1$

103. Solution of the differential equation
$\cos x\, dy = y(\sin x - y)dx$, $0 < x < \pi/2$ is
(a) $\sec x = (\tan x + c)y$ (b) $y \sec x = \tan x + c$
(c) $y \tan x = \sec x + c$ (d) $\tan x = (\sec x + c)y$

104. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is
(a) 2^5 (b) 5^3 (c) 5^2 (d) 3^5

105. If $p = \sin(\cot^{-1}x)$, $q = \cot(\sin^{-1}x)$ and $p^2 = \frac{f(q)}{1+f(q)}$ then $f(x) =$
(a) $x^2 + 1$ (b) $\frac{1}{x^2}$
(c) $x^2 - 1$ (d) none of these

106. If $\sum_{r=1}^{\infty} \cot^{-1}\left\{\frac{2}{x} + \frac{(r^2+r)x}{2}\right\} = \tan^{-1} a$, then $a =$
(a) $x/2$ (b) $2/x$
(c) $2x$ (d) none of these

107. The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point (1, 1) and the coordinate axes, lies in the first quadrant. If its area is 2, then the value of b is
(a) -1 (b) 3 (c) -3 (d) 1

108. If $f(a) = \int_{a-1}^{a+1} \frac{dx}{1+x^8}$, then the value of a for which $f(a)$ attains maximum is
(a) at $a = 0$
(b) at one value of a only
(c) at two values of a one in $(-1,0)$ and the other in $(0,1)$
(d) at no value of a.

109. Let $g(x) = \int_0^x f(t)\, dt$, where f is such that $1/2 \le f(t) \le 1$ for $t \in [0, 1]$ and $0 \le f(t) \le 1/2$ for $t \in [1, 2]$, then $g(2)$ satisfies the inequality
(a) $\frac{1}{2} \le g(2) \le \frac{3}{2}$ (b) $0 \le g(2) < 2$
(c) $\frac{3}{2} < g(2) \le \frac{5}{2}$ (d) $2 < g(2) < 4$

110. Solution of the equation
$x\, dy = \left(y + x\frac{f(y/x)}{f'(y/x)}\right) dx$ is
(a) $f\left(\frac{x}{y}\right) = cy$ (b) $f\left(\frac{y}{x}\right) = cx$
(c) $f\left(\frac{y}{x}\right) = cxy$ (d) $f\left(\frac{y}{x}\right) = 0$

111. Two tailors P and Q earn ₹ 350 and ₹ 450 per day respectively. Tailor P can stitch 6 shirts and 3 trousers while tailor Q can stitch 7 shirts and 3 trousers per day. How many days should each of them work, if it is desired to produce at least 51 shirts and 24 trousers at a minimum labour cost?
(a) Minimize $Z = 350x + 450y$
Subject to, $6x + 7y \ge 51$, $3x + 3y \ge 24$, $x \le 0$, $y \ge 0$
(b) Minimize $Z = 350x + 450y$
Subject to, $6x + 7y \ge 51$, $3x + 3y \ge 24$, $x \ge 0$, $y \ge 0$
(c) Minimize $Z = 350x + 450y$
Subject to, $6x + 7y \ge 51$, $3x + 3y \ge 24$, $x \ge 0$, $y \le 0$
(d) Minimize $Z = 350x + 450y$
Subject to, $6x + 7y \ge 51$, $3x + 3y \ge 24$, $x \le 0$, $y \le 0$

112. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is
(a) $p = q$ (b) $p = 2q$
(c) $q = 2p$ (d) $q = 3p$

113. If the variance of 1, 2, 3, 4, 5, …, 10 is $\frac{99}{12}$, then the standard deviation of 3, 6, 9, 12, …, 30 is
(a) $\frac{297}{4}$ (b) $\frac{3}{2}\sqrt{33}$
(c) $\frac{3}{2}\sqrt{99}$ (d) $\sqrt{\frac{99}{12}}$

114. Mean marks scored by the students of a class is 53. The mean marks of the girls is 55 and the mean marks of the boys is 50. What is the percentage of girls in the class ?
(a) 60% (b) 40% (c) 50% (d) 45%

115. Find the solution set of
$|2\sqrt{2}\sin\theta - \sqrt{3}| \le 1 \text{ in } 0 < \theta < \frac{\pi}{2}$
(a) $\left[\frac{\pi}{12}, \frac{5\pi}{12}\right]$ (b) $\left[\frac{\pi}{16}, \frac{5\pi}{16}\right]$
(c) $\left[\frac{\pi}{10}, \frac{3\pi}{10}\right]$ (d) None of these

116. If $\int_{\pi/e}^{\pi e} \left|\log\frac{x}{\pi}\right| dx = \alpha\left(1 - \frac{1}{e}\right)$, then $\alpha =$
(a) π (b) 2π
(c) $\frac{1}{\pi}$ (d) None of these

117. If $x = 1 + a + a^2 + \ldots \infty$, $y = 1 + b + b^2 + \ldots \infty$, $z = 1 + c + c^2 + \ldots \infty$; a, b, c are in A.P. (where $|a|$, $|b|$, $|c| < 1$) then xy, zx, yz are in
(a) A.P. (b) G.P.
(c) H.P. (d) None of these

118. The area enclosed by $y = |\sin x|, |x| = 2\pi$ and x-axis is
(a) 4 (b) 8π
(c) 8 (d) None of these

119. If $\sum_{r=1}^{\infty} \cot^{-1}\left\{\frac{2}{x} + \frac{(r^2+r)x}{2}\right\} = \tan^{-1} a$, then $a =$
(a) $\frac{x}{2}$ (b) $\frac{2}{x}$
(c) $2x$ (d) None of these

120. If $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$, the modulus and argument form of $(1 + \cos 2\alpha) + i \sin 2\alpha$ is
(a) $-2\cos\alpha[\cos(\pi + \alpha) + i\sin(\pi + \alpha)]$
(b) $2\cos\alpha[\cos\alpha + i\sin\alpha]$
(c) $2\cos\alpha[\cos(-\alpha) + i\sin(-\alpha)]$
(d) $-2\cos\alpha[\cos(\alpha - \pi) + i\sin(\alpha - \pi)]$

121. A quadratic equation $f(x) = ax^2 + bx + c = 0$ $(a \neq 0)$ has positive distinct roots reciprocal of each other. Which one is correct?
(a) $f'(1) = 0$ (b) $af'(1) < 0$
(c) $af'(1) > 0$
(d) nothing can be said about $af'(1)$

122. The period of the function $\sin 3x + \{x\}$ (where $\{\cdot\}$ is fractional part of x) is
(a) $\frac{2\pi}{3}$ (b) 2π
(c) $\frac{\pi}{6}$ (d) does not exist

123. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is
(a) 630 (b) 879 (c) 880 (d) 629

124. If ${}^{12}P_r = {}^{11}P_6 + 6 \cdot {}^{11}P_5$, then $r =$
(a) 7 (b) 5 (c) 6 (d) 4

125. If the product of the roots of the equation $x^2 - 2\sqrt{2}kx + 2e^{2\log k} - 1 = 0$ is 31, then the roots of the equation are real for $k =$
(a) -4 (b) 1 (c) 4 (d) 0

(ENGLISH & LOGICAL REASONING)

126. Complete the sentence : The higher you go, the more difficult it ..... to breathe.
(a) Is becoming (b) Became
(c) Has become (d) Becomes

Direction : *In the following sentence, choose the word opposite in meaning to the bold word to fill in the blanks.*

127. **Absolute** control of the firm is what he wanted, but he ended up with ..... powers.
(a) complex (b) limited
(c) little (d) few

Pick up the correct synonym

128. Timid
(a) Veteran (b) Fearful
(c) Cowardly (d) Plucky

Direction: *In the given question, out of the four alternatives, choose the one which can be substituted for the given words/sentence.*

129. A light sailing boat built especially for racing
(a) Yacht (b) Frigate
(c) Dinghy (d) Canoe

Direction : *In the given question, a word has been written in four different ways out of which only one is correctly spelt. Find the correctly spelt word.*

130. (a) Temperature (b) Tamperature
(c) Tempereture (d) Temparature

Direction : *In the following question, find out which part of the sentence has an error.*

131. If you are great at ideas but not very good at getting into
(a) / the nitty gritty
(b) / of things and implementing them, then you work on a team
(c) / that has someone who can implement
(d)

Directions (Question 132 – 134) : *Read the passage and answer the following questions.*

The low unit of gas is a real temptation to anyone choosing between gas and electrical processes. But gas-fired processes are often less efficient, require more floor space, take longer and produce more variable product quality. The drawbacks negate the savings many businesses believe they make.

By contrast, electricity harnesses a unique range of technologies unavailable with gas. And many electric processes are well over 90 percent efficient, so far less energy is wasted with benefits in terms of products quality and overall cleanliness, it can so often be the better and cheaper choice. Isn't that tempting?

132. The passage can be described as
(a) An advertisement for electricity and its efficiency
(b) An extract from a science journal
(c) An account of the growth of technology
(d) An appeal not to use gas.

133. What does the writer mean by 'variable quality'?
(a) The quality of the products cannot be assessed
(b) Products from gas-fired processes are inefficient
(c) The kind of products vary from time to time
(d) The quality of the products is not uniform.

134. "Electricity harnesses a unique range of technologies" - What does the writer mean? Electricity
(a) Has developed new technologies
(b) Ensures power for electricity and its efficiency
(c) Depends on new kinds of technology
(d) Makes use of several technologies.

Direction : *In the following question, choose the alternative which can replace the word printed in bold without changing the meaning of the sentence.*

135. A bone got stuck in his gullet.
(a) Chest (b) Throat
(c) Stomach (d) Molars

Directions (Question 136 – 140) : *In each of the following questions, a sentence is given with a blank to be filled in with an appropriate word. Four alternatives are suggested for each question. Choose the correct alternative.*

136. He did not register his ..... to the proposal.
(a) Disfavour (b) Dissent
(c) Deviation (d) Divergence

137. Will you, like the ..... gentleman and solider you are, leave at once before he finds you here?
(a) Chivalrous (b) Luminous
(c) Barbarous (d) Ostentatious

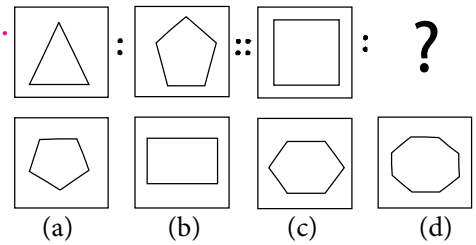
138. In these days of inflation, the cost of consumer goods is .....
(a) Climbing (b) Raising
(c) Ascending (d) Soaring

139. The Committee's appeal to the people for money ..... little response.
(a) Gained (b) Provided
(c) Evoked (d) Provoked

140. The manager tried hard to ..... his men to return to work before declaring a lockout.
(a) Encourage (b) Permit
(c) Motivate (d) Persuade

Direction : *In the question, two figures are given to the left of the sign : : and one figure to the right of the sign : : with four alternatives under it out of which one of the alternatives has the same relationship with the figure to the right of the sign : : as between the two figures to the left of the sign* **(: :).** *Find the correct alternative.*

141.



(a) (b) (c) (d)

Direction : *In the following question, three words are given. They are followed by four words one of which stands for the class to which these three words belong. Identify that word.*

142. Instill, Inside, Inform
(a) Intent (b) Interview
(c) Intelligent (d) Institute

Direction : *In the following question a word has been given followed by four other words, one of which cannot be formed by using the letters of the given word. Find this word.*

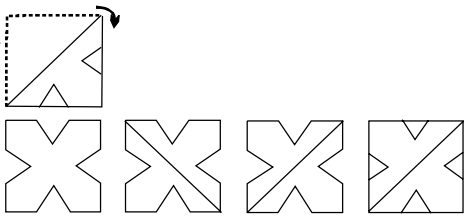
143. INTELLIGENCE

(a) NEGLECT (b) GENTLE

(c) INCITE (d) CANCEL

Direction : *In the following question a piece of paper is folded as shown below and a cut is made as marked. How would the paper look like when unfolded?*

144.



(a) (b) (c) (d)

Direction : *In the given letter sequence some letters are missing which are given in that order as one of the four alternatives under it. Find out the correct one from the responses* (a), (b), (c), *and* (d) *given under it.*

145. b a—b a—b a c—a c b—c b a c

(a) a a c b (b) b b c a

(c) c c b a (d) c b a c

Direction : *Complete the following series.*

146. 6 8 9 12 14 18 ?

(a) 19 (b) 21

(c) 23 (d) 25

Direction : *Find odd one out.*

147. (a) MNW (b) OPQ

(c) ILT (d) GHC

Direction : *One of the words given in the alternatives cannot be formed by using the letters of the given word in the question. Find out that word.*

148. TEACHERS

(a) REACH (b) SEARCH

(c) CHAIR (d) CHEER

Direction : *Find out the number from amongst the four alternatives that can replace the question mark (?) as given in a cell of the matrix.*

149.

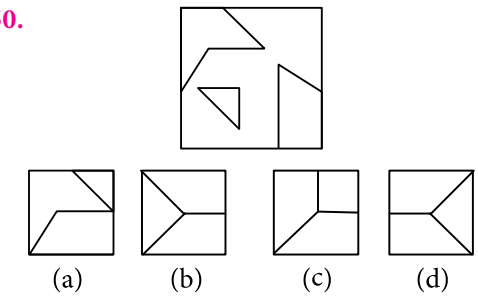
?	1	2
21	22	40
1	2	5
20	23	43

(a) 2 (b) 3

(c) 4 (d) 5

Direction: *In following question you have to find out from amongst the four answer figures that can be formed from the cut-out pieces given in the question.*

150.



(a) (b) (c) (d)

ANSWER KEYS

1.	(d)	**2.**	(c)	**3.**	(b)	**4.**	(b)	**5.**	(c)
6.	(a)	**7.**	(b)	**8.**	(c)	**9.**	(a)	**10.**	(c)
11.	(b)	**12.**	(a)	**13.**	(a)	**14.**	(b)	**15.**	(c)
16.	(a)	**17.**	(a)	**18.**	(b)	**19.**	(b)	**20.**	(c)
21.	(b)	**22.**	(a)	**23.**	(c)	**24.**	(b)	**25.**	(c)
26.	(d)	**27.**	(a)	**28.**	(b)	**29.**	(a)	**30.**	(d)
31.	(c)	**32.**	(b)	**33.**	(d)	**34.**	(a)	**35.**	(b)
36.	(a)	**37.**	(a)	**38.**	(b)	**39.**	(d)	**40.**	(d)
41.	(a)	**42.**	(d)	**43.**	(c)	**44.**	(b)	**45.**	(d)
46.	(b)	**47.**	(a)	**48.**	(c)	**49.**	(c)	**50.**	(a)
51.	(b)	**52.**	(a)	**53.**	(d)	**54.**	(d)	**55.**	(c)
56.	(c)	**57.**	(a)	**58.**	(c)	**59.**	(a)	**60.**	(c)
61.	(d)	**62.**	(b)	**63.**	(c)	**64.**	(a)	**65.**	(a)
66.	(a)	**67.**	(d)	**68.**	(b)	**69.**	(d)	**70.**	(d)
71.	(a)	**72.**	(d)	**73.**	(b)	**74.**	(a)	**75.**	(b)
76.	(a)	**77.**	(a)	**78.**	(d)	**79.**	(b)	**80.**	(a)
81.	(b)	**82.**	(c)	**83.**	(a)	**84.**	(d)	**85.**	(a)
86.	(a)	**87.**	(b)	**88.**	(b)	**89.**	(a)	**90.**	(c)
91.	(c)	**92.**	(c)	**93.**	(a)	**94.**	(a)	**95.**	(c)
96.	(c)	**97.**	(c)	**98.**	(a)	**99.**	(b)	**100.**	(b)
101.	(c)	**102.**	(a)	**103.**	(a)	**104.**	(d)	**105.**	(a)
106.	(b)	**107.**	(c)	**108.**	(a)	**109.**	(a)	**110.**	(b)
111.	(b)	**112.**	(d)	**113.**	(b)	**114.**	(a)	**115.**	(a)
116.	(b)	**117.**	(a)	**118.**	(c)	**119.**	(b)	**120.**	(d)
121.	(b)	**122.**	(d)	**123.**	(b)	**124.**	(c)	**125.**	(c)
126.	(d)	**127.**	(b)	**128.**	(c)	**129.**	(a)	**130.**	(a)
131.	(c)	**132.**	(b)	**133.**	(d)	**134.**	(b)	**135.**	(b)
136.	(b)	**137.**	(a)	**138.**	(d)	**139.**	(c)	**140.**	(d)
141.	(c)	**142.**	(a)	**143.**	(d)	**144.**	(a)	**145.**	(c)
146.	(b)	**147.**	(c)	**148.**	(c)	**149.**	(a)	**150.**	(a)

EXAMINATION DATES 2015

4 April	JEE (Main-Offline)	29-30 April	Karnataka CET
10 – 11 April	JEE (Main-Online)	3 May	AIPMT
8 – 19 April	VITEEE	10 May	COMED K
18-19 April	WB JEE	16 May	UPSEAT
19 April	MGIMS	24 May	JEE Advanced
20-21April	Kerala PET	1 June	AIIMS
22-23 April	Kerala PMT	7 June	JIPMER

PRACTICE PAPER

for ONLINE TEST

JEE MAIN 2015

Exam on 10th &11th April

1. If $\alpha \neq 1, \alpha^6 = 1$ and $\sum_{r=1}^{6} {}^6C_r . \alpha^{r-1} = x$, then the value of $|x|$ =
(a) 22 (b) 24 (c) 26 (d) 28

2. Let $f(x) = x^2 - bx + c$, b is an odd positive integer, $f(x) = 0$ has two prime numbers as roots and $b + c = 35$. Then the global minimum value of the function $f(x)$ is
(a) $-\frac{183}{4}$ (b) $\frac{173}{16}$ (c) $-\frac{81}{4}$ (d) $\frac{81}{4}$

3. Maximum value of $\log_5(3x + 4y)$, if $x^2 + y^2 = 25$ is given by
(a) 2 (b) 3 (c) 4 (d) 5

4. **Statement-1 :** $[\vec{a}\,\vec{b}\,\vec{c}]\vec{r} = [\vec{r}\,\vec{b}\,\vec{c}]\vec{a} + [\vec{r}\,\vec{c}\,\vec{a}]\vec{b} + [\vec{r}\,\vec{a}\,\vec{b}]\vec{c}$

Statement-2 : $(\vec{r} \times \vec{a}) \times (\vec{b} \times \vec{c}) = [\vec{r}\,\vec{a}\,\vec{c}]\vec{b} - [\vec{r}\,\vec{a}\,\vec{b}]\vec{c}$.

Also $(\vec{r} \times \vec{a}) \times (\vec{b} \times \vec{c}) = [\vec{r}\,\vec{b}\,\vec{c}]\vec{a} - [\vec{a}\,\vec{b}\,\vec{c}]\vec{r}$

(a) Statement-1 is true, statement-2 is true: statement-2 is a correct explanation for statement-1
(b) Statement-1 is true, statement-2 is true: statement-2 is NOT a correct explanation for statement-1
(c) Statement-1 is true, statement-2 is false
(d) Statement-1 is false, statement-2 is true

5. The coefficient of x^6 in the expansion of

$$\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}\right)^2$$ is given by

(a) $\frac{2}{15}$ (b) $\frac{4}{15}$ (c) $\frac{31}{360}$ (d) $\frac{2}{45}$

6. If $f: R \to R$ is defined by $f(x) = x - [x] - \frac{1}{2}\ \forall\ x \in R$, where $[x]$ denotes the greatest integer function, then solution set of $f(x) = 1/2$ is given by
(a) Z, the set of all integers
(b) N, the set of all natural numbers
(c) ϕ, the empty set
(d) R, the set of all real numbers

7. Consider a triangle with sides 3 cm, 6 cm, 8 cm respectively. Now if a man runs around the triangle in such a way that he is always at a distance of 1 cm from the side of triangle, then distance travelled by him is given by
(a) 17 (b) 20
(c) $17 + 2\pi$ (d) $17 + 3\pi$

8. The number of points at which the function $f(x) = (x - |x|)^2 (1 - x + |x|)^2$ is not differentiable in the interval $(-3, 4)$ is
(a) 0 (b) 1 (c) 2 (d) 3

9. The angle between a pair of tangents drawn to the curve $7x^2 - 12y^2 = 84$ from $M(1, 2)$ is given by
(a) $\pi/4$ (b) $\pi/2$ (c) $\pi/6$ (d) $\pi/3$

10. **Statement-1 :** If $f(x) = \begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$;
where ω is a complex cube root of unity. Then $\int_{-\pi/2}^{\pi/2} f(x)dx = 0$.

Statement-2 : $\int_0^{2a} f(x)dx = 0$, if $f(2a - x) = -f(x)$, where $f(x)$ is a function of the real variable x.
(a) Statement-1 is true, statement-2 is true: statement-2 is a correct explanation for statement-1
(b) Statement-1 is true, statement-2 is true: statement-2 is NOT a correct explanation for statement-1
(c) Statement-1 is true, statement-2 is false
(d) Statement-1 is false, statement-2 is true

By : Alok Sir
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11. The position vector of a point in which a line through the origin perpendicular to the plane $2x - y - z = 4$ meet the plane $\vec{r} \cdot (3\hat{i} - 5\hat{j} + 2\hat{k}) = 6$, is

(a) $(1, -1, -1)$ (b) $(-1, -1, 2)$

(c) $(4, 2, 2)$ (d) $\left(\frac{4}{3}, -\frac{2}{3}, -\frac{2}{3}\right)$

12. If $f(x) = \sqrt[3]{\frac{9}{\log_2(3-2x)} - 1}$, then the value of a which satisfies $f^{-1}(2a - 4) = 1/2$, is given by

(a) 1 (b) 2 (c) 3 (d) 4

13. Let S be an interval in which $f(x) = \int_0^x \frac{dt}{1+t^3}$ is an invertible function. Suppose g is the inverse of f, then which of the following statement is true?

(a) $g'(x) = \frac{1}{f(f'(x))}$ (b) $g'(x) = \frac{1}{g(g'(x))}$

(c) $g'(x) = \frac{1}{g(f'(x))}$ (d) $g'(x) = \frac{1}{f'(g(x))}$

14. **Statement-1 :** The greatest value of the function given by $f(x) = \begin{cases} 2x+2; & -4 \le x < 1 \\ 4x^2 - 16x + 16; & 1 \le x \le 4 \end{cases}$, is 4.

Statement-2 : A function which is not differentiable at a point can have an extreme value at that point.

(a) Statement-1 is true, statement-2 is true: statement-2 is a correct explanation for statement-1

(b) Statement-1 is true, statement-2 is true: statement-2 is NOT a correct explanation for statement-1

(c) Statement-1 is true, statement-2 is false

(d) Statement-1 is false, statement-2 is true

15. If the line $\frac{x-2}{3} = \frac{y-1}{5} = \frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$. Then the value of $\alpha + \beta$ is

(a) 0 (b) 1 c) 2 (d) -3

16. Let f and g are the two functions symmetric about the line $x = a$ and $h(x)$ be a function such that $3h(x) - 4h(2a - x) = 5$. Then the value of $\frac{\int_0^{2a} f(x) \cdot g(x) \cdot h(x)\, dx}{\int_0^{2a} f(x) \cdot g(x)\, dx}$ is

(a) -2 (b) 5 (c) 3 (d) -5

17. Let a relation R in the set N of natural numbers be defined as $(x, y) \in R$ if and only if $x^2 - 4xy + 3y^2 = 0$ for all $x, y \in N$. Then the relation R is

(a) Reflexive (b) Symmetric

(c) Transitive (d) Equivalence

18. **Statement-1 :** The number of symmetric relations on a set with 3 elements is 64.

Statement-2 : If a set has n elements, then the number of symmetric relations on A is $2^{n^2} - n$.

(a) Statement-1 is true, statement-2 is true: statement-2 is a correct explanation for statement-1

(b) Statement-1 is true, statement-2 is true: statement-2 is NOT a correct explanation for statement-1

(c) Statement-1 is true, statement-2 is false

(d) Statement-1 is false, statement-2 is true

19. If $\int_0^{n\pi+t} |\sin x|\, dx = k - \cos t\ (0 \le t \le \pi)$, then the value of k is

(a) $2n$ (b) $4n + 1$ (c) $3n + 1$ (d) $2n + 1$

20. Number of solutions of the equation $[\cos x] = \{\sin\{x\}\}$ in $[0, 2\pi]$ are

(a) 0 (b) 2 (c) 3 (d) 4

21. If $\alpha = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then $B = A + A^2 + A^3 + A^4$ is

(a) Singular (b) Non-singular

(c) Symmetric (d) $|B| = 1$

22. Let p, q, r be three statements.

Statement-1 : $(p \to q) \wedge (q \to r) \leftrightarrow (p \to r)$.

Statement-2 : If $p \to q$ and $q \to r$, then $p \to r$ is a tautology.

(a) Statement-1 is true, statement-2 is true: statement-2 is a correct explanation for statement-1

(b) Statement-1 is true, statement-2 is true: statement-2 is NOT a correct explanation for statement-1

(c) Statement-1 is true, statement-2 is false

(d) Statement-1 is false, statement-2 is true

23. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ and $f(0) = 1$ and g be a function satisfying $f(x) + g(x) = x^2$. If the value of $\int_0^1 f(x)g(x)\, dx - e + \frac{e^2}{2}$ is k, then the value of $|2k|$ is

(a) 1 (b) 2 (c) 3 (d) 4

24. The intercept of the common tangent to the curves $y^2 = 8x$ and $xy + 1 = 0$ on the axis of x is
(a) -1 (b) -2 (c) 1 (d) 2

25. Two man A and B fire at a target. Suppose $P(A) = 1/3$ and $P(B) = 1/5$ are the probabilities of hitting the target by A and B respectively. The probability that the target got hit, is
(a) 1/15 (b) 6/15 (c) 7/15 (d) 8/15

26. Two points A and B are taken on the curve $y = \ln x$ whose abscissa are 1 and e respectively. The chord AB is formed. A point on the curve at which the tangent line is parallel to the chord AB is
(a) $(e + 1, \ln(e + 1))$ (b) $(2, \ln 2)$
(c) $(e - 2, \ln(e - 2))$ (d) $(e - 1, \ln(e - 1))$

27. The cubic equation $4x^3 - 33x^2 + 72x + 6 = 0$ has
(a) one positive real root and two complex roots
(b) one negative real root and two complex roots
(c) all three real roots, one negative and two positive
(d) all three real roots, one positive and two negative

28. Let $f(x) = x^4 e^{-x^2}$, $\forall x \in R$. Then $f(x)$ has
(a) Maxima at $x = 0$ (b) Minima at $x = \pm 2$
(c) Minima at $x = \pm 1$ (d) Maxima at $x = \pm\sqrt{2}$

29. Consider a quadratic equation given by $\sum_{r=1}^{n}\left(3x - \frac{1}{2r+1}\right)\left(2x - \frac{1}{2r+5}\right) = 0$. The product of roots of the equation is
(a) $\frac{n+1}{6n(2n+5)}$ (b) $\frac{n+1}{36(2n+5)}$
(c) $\frac{n+1}{36n(2n+5)}$ (d) $\frac{n+1}{72n(2n+5)}$

30. Let $f: R \to (-1, 1)$ defined by $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. Then f is
(a) one-one onto
(b) one-one but not onto
(c) onto but not one-one
(d) Neither one-one nor onto

SOLUTIONS

1. (d) : As, $\alpha^6 = 1$

$\Rightarrow \alpha = \left(\cos\frac{2\pi}{6} + i\sin\frac{2\pi}{6}\right)^m$, where $m \to 0$ to 5

Taking any one of its roots,

$\alpha = \cos\frac{2\pi}{6} + i\sin\frac{2\pi}{6}$

Now, $\sum_{r=1}^{6} {}^6C_r \cdot \alpha^r = \alpha x$

$\Rightarrow \alpha x = (1+\alpha)^6 - 1 = \left\{2\cos\frac{\pi}{6}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right\}^6 - 1$

$= -27 - 1 = -28$

$\Rightarrow |\alpha x| = 28 \Rightarrow |x| = 28$

2. (c): Let α, β be the roots of $x^2 - bx + c = 0$, then $\alpha + \beta = b$

$\Rightarrow$ One root is 2, as roots are prime and b is odd integer

[$\because$ α and β both simultaneously can't be odd, so one root must be odd and other even]

$\therefore$ $f(2) = 0$

$\Rightarrow 2b - c = 4$ and $b + c = 35$ (given)

So, $b = 13$, $c = 22$

$\therefore$ Minimum value $= f\left(\frac{13}{2}\right) = -\frac{81}{4}$

3. (a) : Let $x = 5\cos\theta$, $y = 5\sin\theta$

So, given expression $= \log_5\{5(3\cos\theta + 4\sin\theta)\}$

As $3\cos\theta + 4\sin\theta \le 5$

$\Rightarrow$ Maximum value $= \log_5(25) = 2$

4. (a) : Statement-2 : Put $\vec{r} \times \vec{a} = \vec{m}$

$\Rightarrow \vec{m} \times (\vec{b} \times \vec{c}) = (\vec{m} \cdot \vec{c})\vec{b} - (\vec{m} \cdot \vec{b})\vec{c}$

$= [\vec{r}\ \vec{a}\ \vec{c}]\vec{b} - [\vec{r}\ \vec{a}\ \vec{b}]\vec{c}$ ... (i)

Also, if $\vec{b} \times \vec{c} = \vec{n}$

$(\vec{r} \times \vec{a}) \times \vec{n} = (\vec{r} \cdot \vec{n})\vec{a} - (\vec{a} \cdot \vec{n})\vec{r}$

$= [\vec{r}\ \vec{b}\ \vec{c}]\vec{a} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{r}$ ... (ii)

Statement-2 is true

From (i) and (ii)

$[\vec{r}\ \vec{a}\ \vec{c}]\vec{b} - [\vec{r}\ \vec{a}\ \vec{b}]\vec{c} = [\vec{r}\ \vec{b}\ \vec{c}]\vec{a} - [\vec{r}\ \vec{b}\ \vec{c}]\vec{r}$

$\Rightarrow [\vec{r}\ \vec{b}\ \vec{c}]\vec{r} = [\vec{r}\ \vec{b}\ \vec{c}]\vec{a} + [\vec{r}\ \vec{a}\ \vec{b}]\vec{c} - [\vec{r}\ \vec{a}\ \vec{c}]\vec{b}$

$= [\vec{r}\ \vec{b}\ \vec{c}]\vec{a} + [\vec{r}\ \vec{a}\ \vec{b}]\vec{c} + [\vec{r}\ \vec{c}\ \vec{a}]\vec{b}$

$\Rightarrow$ Statement-1 is also correct and statement-2 is correct explanation of statement-1

5. (c): Required coefficient of x^6 is given by

$2\left\{\frac{1}{1! \times 5!} + \frac{1}{2! \times 4!}\right\} + \frac{1}{(3!)^2}$

$= \frac{1}{6!}\{{}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5\} = \frac{2^6 - 2}{6!} = \frac{31}{360}$

6. (c) : As $f(x) = 1/2$

$\Rightarrow x - [x] = 1$, which is not true

as $x - [x] \in [0, 1)$

i.e., given set is the empty set.

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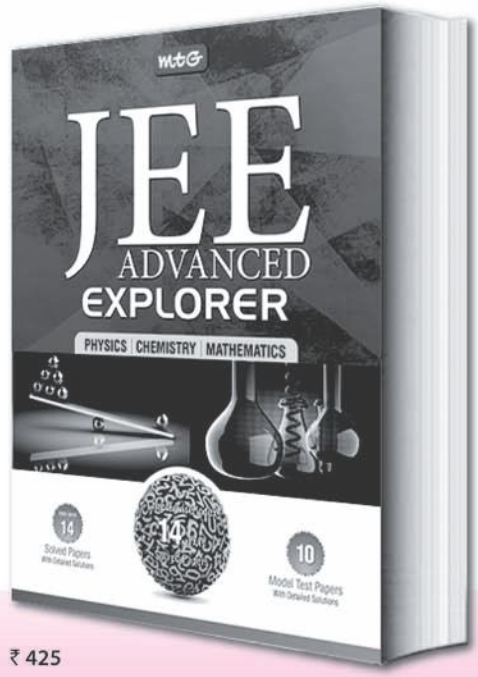

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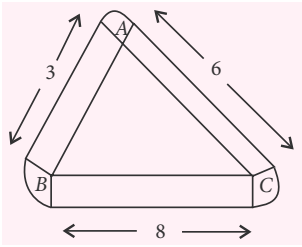
7. (c):



Distance travelled by man is

$3 + 6 + 8 + (\pi - A) \times 1 + (\pi - B) \times 1 + (\pi - C) \times 1$

$= 17 + 3\pi - (A + B + C) = 17 + 2\pi$

8. (a) : Given $f(x)$ can be written as

$$f(x) = \begin{cases} 16x^4 - 16x^3 + 4x^2, & x < 0 \\ 0, & x \geq 0 \end{cases}$$

Clearly, $f(x)$ is continuous as well as derivable $\forall\, x \in R$.

9. (b) : The equation of pair of tangents from (x_1, y_1) on curve $\equiv SS_1 = T^2$

$$\therefore \left(\frac{x^2}{12} - \frac{y^2}{7} - 1\right)\left(\frac{1^2}{12} - \frac{2^2}{7} - 1\right) = \left(\frac{1 \cdot x}{12} - \frac{y(2)}{7} - 1\right)^2$$

$$\Rightarrow \left(\frac{x^2}{12} - \frac{y^2}{7} - 1\right)\left(\frac{7 - 48 - 84}{84}\right) = \left(\frac{7x - 24y - 84}{84}\right)^2$$

On solving, we have

$(7x^2 - 12y^2 - 84)(7 - 48 - 84) = (7x - 24y - 84)^2$

$\Rightarrow 924x^2 - 924y^2 - 336xy + \alpha x + \beta y + c = 0$

$\because$ Coefficient of x^2 + coefficient of $y^2 = 0$

$\therefore$ Angle subtended is $\pi/2$

10. (b) : Statement-2 is the standard result

$$f(x) = \begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

$$C_1 \to C_1 + C_2 + C_3$$

$$\Rightarrow f(x) = x\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & x+\omega^2 & 1 \\ 1 & 1 & x+\omega \end{vmatrix}$$

$$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$$

$$f(x) = x\begin{vmatrix} 1 & \omega & \omega^2 \\ 0 & x+\omega^2-\omega & 1-\omega^2 \\ 0 & 1-\omega & x+\omega-\omega^2 \end{vmatrix}$$

$= x[(x + \omega^2 - \omega)(x + \omega - \omega^2) - (1 - \omega^2)(1 - \omega)]$

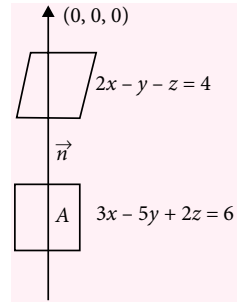
$= x[x^2 - (\omega + \omega^2 - 2) - (1 - \omega - \omega^2 + 1)] = x^3$

$\Rightarrow$ $f(x)$ is an odd function.

$$\therefore \int_{-\pi/2}^{\pi/2} f(x)dx = 0$$

Both statements are correct but statement-2 is not the correct explanation of statement-1

11. (d) :



Vector perpendicular to $2x - y - z = 4$ is given by

$\vec{n} = 2\hat{i} - \hat{j} - \hat{k}$

Also, line is collinear with $\vec{n}$.

So, equation of line is

$\vec{r} = \lambda(2\hat{i} - \hat{j} - \hat{k})$ ...(i)

Now, line (i) meets the plane

$\vec{r} \cdot (3\hat{i} - 5\hat{j} + 2\hat{k}) = 6$

So, $\lambda(6 + 5 - 2) = 6 \Rightarrow \lambda = 2/3$

Hence, P.V. of the point A is $\frac{2}{3}(2\hat{i} - \hat{j} - \hat{k})$

i.e. $\left(\frac{4}{3}, \frac{-2}{3}, \frac{-2}{3}\right)$

12. (c): Given $f^{-1}(2a - 4) = \frac{1}{2}$

$$\therefore f\left(\frac{1}{2}\right) = 2a - 4$$

Put $x = 1/2$ in $f(x) = \sqrt[3]{\frac{9}{\log_2(3 - 2x)} - 1}$

We have, $f\left(\frac{1}{2}\right) = \sqrt[3]{9 - 1} = 2$

$\therefore$ $2a - 4 = 2 \Rightarrow a = 3$

13. (d) : $\because$ $f'(x) = \frac{1}{1 + x^3}$

Also, $f(g(x)) = \int_0^{g(x)} \frac{dt}{1 + t^3}$ $(\because\ g = f^{-1})$

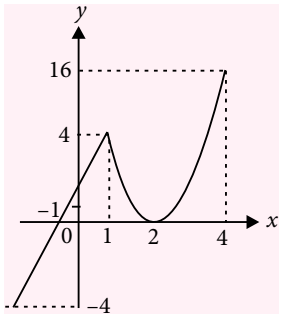
$$\Rightarrow x = \int_0^{g(x)} \frac{dt}{1 + t^3}$$

Now, $1 = \frac{1}{1 + (g(x))^3} g'(x)$

$\Rightarrow g'(x) = 1 + (g(x))^3$

$$= \frac{1}{f'(g(x))}$$

14. (d) :



$$f(x)=\begin{cases}2(x+1), & -4\le x<1\\ 4(x-2)^2, & 1\le x\le 4\end{cases}$$

From figure, it is clear that $f(x)$ has a local maxima at $x=1$, but the greatest value is at $x=4$ (at end point) and the greatest value is 16.

15. (b) : The required conditions are

$3-15-2\alpha=0$ ... (i)

and $2+3+2\alpha+\beta=0$ ... (ii)

$\Rightarrow \alpha=-6$ and $\beta=7$

$\therefore \alpha+\beta=1$

16. (d) : $\because f(a-x)=f(a+x)$

or, $f(x+2a)=f(-x) \Rightarrow f(x)=f(2a-x)$

Similarly, $g(2a-x)=g(x)$

Now, let $I=\int_0^{2a} f(x)\cdot g(x)\cdot h(x)dx$

$$=\int_0^{2a} f(x)\cdot g(x)\cdot h(2a-x)dx$$

or, $4I=\int_0^{2a} f(x)\cdot g(x)\cdot(3h(x)-5)dx$

$$=3I-5\int_0^{2a} f(x)\cdot g(x)dx$$

$\therefore I=-5\int_0^{2a} f(x)\cdot g(x)dx$

$$\Rightarrow \frac{\int_0^{2a} f(x)\cdot g(x)\cdot h(x)dx}{\int_0^{2a} f(x)\cdot g(x)dx}=-5$$

17. (a)

18. (c): Let $A=\{x_1, x_2, \ldots, x_n\}$

$A\times A=\{(x_1,x_1),(x_1,x_2),\ldots,(x_2,x_1),(x_2,x_2),\ldots\}$

For symmetric relations if

$(a,b)\in R \leftrightarrow (b,a)\in R$

The elements of type (a,a)

i.e., $(x_1,x_1),(x_2,x_2),\ldots,(x_n,x_n)$

$\Rightarrow$ n elements

Remaining elements in $A\times A=n^2-n$

$\Rightarrow$ The number of symmetric relations

$$={}^{\lambda}C_0+{}^{\lambda}C_1+\ldots.+{}^{\lambda}C_{\lambda}=2^{\lambda}$$

where $\lambda=n+\frac{n^2-n}{2}=\frac{n^2+n}{2}$

$\therefore$ The number of symmetric relations $=2^{\frac{n^2+n}{2}}$

$\Rightarrow$ Statement-2 is false

If $n(A)=3$, then the number of symmetric relations

$$=2^{\frac{9+3}{2}}=2^6=64$$

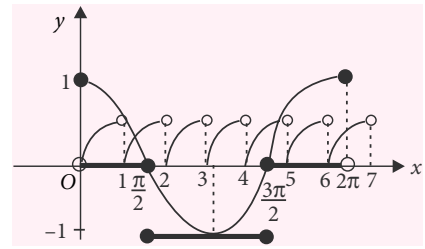
19. (d) : $\int_0^{n\pi+t}|\sin x|dx=\int_0^{n\pi}|\sin x|dx+\int_{n\pi}^{n\pi+t}|\sin x|dx$

$$=2n+(-\cos x)_{n\pi}^{n\pi+t}$$

$$=2n+\cos(n\pi)-\cos(n\pi+t)$$

$$=\begin{cases}2n+1-\cos t; & n=\text{even}\\ 2n-1+\cos t; & n=\text{odd}\end{cases}$$

20. (c): From graph it can be seen that number of solutions is 3.



21. (b) : $\because B=A+A^2+A^3+A^4$

$$=\frac{A(I-A^4)}{I-A}=\frac{A-A^5}{I-A}$$

Now, $A^5=\begin{bmatrix}\cos 5\alpha & \sin 5\alpha\\ -\sin 5\alpha & \cos 5\alpha\end{bmatrix}$

$$=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}=-I$$

22. (d) :

p	q	r	$(p\to q)$	$(q\to r)$	$(p\to r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

23. (c): $\because f'(x) = f(x)$ and $f(0) = 1$

$\Rightarrow f(x) = e^x$

$\therefore g(x) = x^2 - e^x$

Now, $\int_0^1 f(x)g(x)\,dx = \int_0^1 e^x(x^2 - e^x)dx = e - \frac{e^2}{2} - \frac{3}{2}$

$\Rightarrow \int_0^1 f(x)g(x)dx - e + \frac{e^2}{2} = \frac{-3}{2}$

$\Rightarrow k = -\frac{3}{2} \Rightarrow |2k| = 3$

24. (b) : Equation of tangent of $y^2 = 8x$ is

$$y = mx + \frac{2}{m} \quad \text{... (i)}$$

and $xy + 1 = 0$ ... (ii)

From (i) and (ii)

$$x\left(mx + \frac{2}{m}\right) + 1 = 0 \quad \text{or} \quad mx^2 + \frac{2}{m}x + 1 = 0$$

$\Rightarrow D = 0$

$\Rightarrow \frac{4}{m^2} = 4m \Rightarrow m^3 = 1$

$\therefore m = 1$

$\therefore$ common tangent is $y = x + 2$

$\therefore$ x-intercept $= -2$

25. (c): $P(E) = P(A)P(B') + P(A')P(B) + P(A)\cdot P(B)$

$$= \left(\frac{1}{3}\times\frac{4}{5}\right) + \left(\frac{2}{3}\times\frac{1}{5}\right) + \left(\frac{1}{3}\times\frac{1}{5}\right) = \frac{4}{15} + \frac{2}{15} + \frac{1}{15} = \frac{7}{15}$$

26. (d) : Using LMVT,

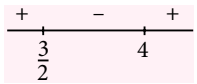
$$f'(c) = \frac{f(b) - f(a)}{b - a} \Rightarrow \frac{1}{c} = \frac{1-0}{e-1}$$

$\therefore$ The point on the curve is $(e - 1, \ln(e - 1))$

27. (b) : $f(x) = 4x^3 - 33x^2 + 72x + 6$

$\Rightarrow f'(x) = 12x^2 - 66x + 72$

$= 6(2x^2 - 11x + 12) = 6(2x - 3)(x - 4)$



$\Rightarrow f(x)$ is increasing in $(-\infty, 3/2) \cup (4, \infty)$ and decreasing in $(3/2, 4)$

$\Rightarrow f(4) = 256 - 528 + 288 + 6 > 0$

$\Rightarrow f\left(\frac{3}{2}\right) = \frac{27}{2} - \frac{297}{4} + 108 + 9 > 0$

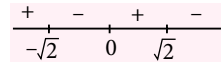
Also, $f(0) = 6 > 0$

$\Rightarrow f(x)$ has one negative real root and other two roots are complex.

28. (d) : $f(x) = x^4 e^{-x^2}$

$\Rightarrow f'(x) = 4x^3 e^{-x^2} - 2x^5 e^{-x^2}$

$\Rightarrow 2x^3 e^{-x^2}(2 - x^2) = 0$



$\Rightarrow x = 0, \pm\sqrt{2}$

$\therefore f(x)$ has maxima at $x = \pm\sqrt{2}$ and minima at $x = 0$

29. (c): Let $\frac{1}{2r+1} = \lambda$ and $\frac{1}{2r+5} = \mu$

$\Rightarrow \sum_{r=1}^{n} (3x - \lambda)(2x - \mu) = 0$

$\Rightarrow \sum_{r=1}^{n} 6x^2 - x(3\mu + 2\lambda) + \lambda\mu = 0$

Since $\sin\mu \sum_{r=1}^{n} \lambda\mu = \sum_{r=1}^{n} \frac{1}{(2r+1)(2r+5)}$

$$\Rightarrow \frac{1}{4}\sum_{r=1}^{n}\left(\frac{1}{2r+1} - \frac{1}{2r+5}\right)$$

$$= \frac{1}{4}\left[\frac{1}{3} - \frac{1}{2n+5}\right] = \frac{n+1}{6(2n+5)}$$

$\therefore$ The product of roots of the equation $= \frac{n+1}{36n(2n+5)}$

30. (a) : $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$$f'(x) = \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2}$$

$$= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2}$$

$$= \frac{2e^x \cdot 2e^{-x}}{(e^x + e^{-x})^2} = \frac{4}{(e^x + e^{-x})^2} > 0, \forall\, x \in R$$

$\Rightarrow f$ is increasing function for all x in R

$\Rightarrow f$ is one-one.

As $x \to \infty$, $f(x) = \frac{1 - e^{-2x}}{1 + e^{-2x}} \to 1$

and $x \to -\infty$, $f(x) = \frac{e^{2x} - 1}{e^{2x} + 1} \to -1$

$\therefore$ Range is $(-1, 1) \Rightarrow f$ is onto.

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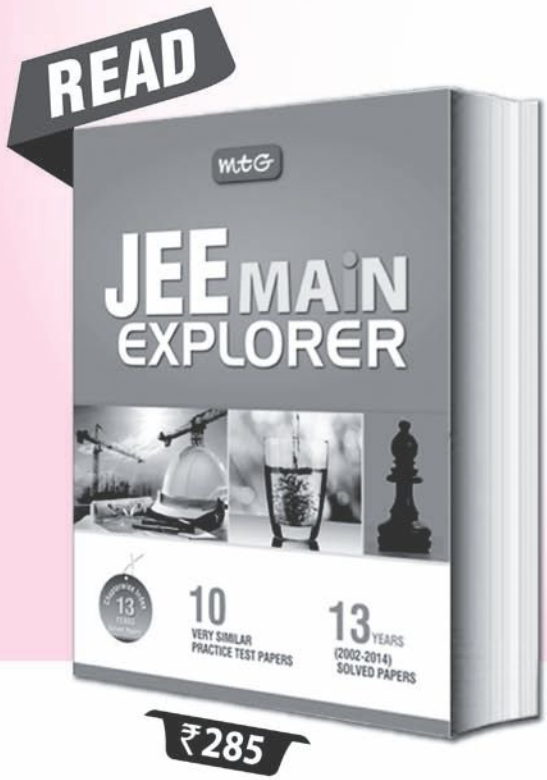

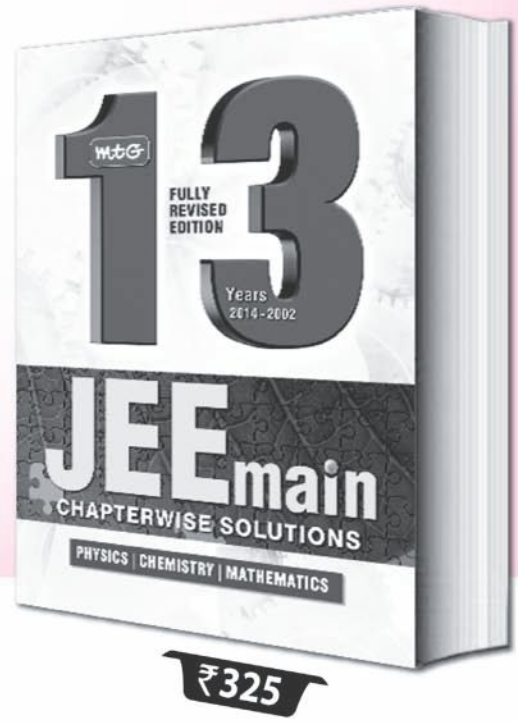

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MATHS MUSING

SOLUTION SET-147

1. (d): $\Delta = (a - b + c)(a + b - c) = 4(s - b)(s - c)$

$\tan\frac{A}{2} = \frac{(s-b)(s-c)}{\Delta} = \frac{1}{4}$, $\sin A = \frac{8}{17}$

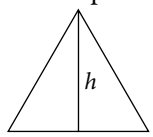
2. (a): $y = \frac{x}{1-x^4} = \frac{-1}{2}\left[\frac{x}{x^2-1} - \frac{x}{x^2+1}\right]$

$= -\frac{1}{4}\left[\frac{1}{x-1} + \frac{1}{x+1} - \frac{1}{x-i} - \frac{1}{x+i}\right]$

$\frac{d^5y}{dx^5} = \frac{5!}{4}\left[\frac{1}{(x-1)^6} + \frac{1}{(x+1)^6} - \frac{1}{(x-i)^6} - \frac{1}{(x+i)^6}\right]$

$\frac{d^5y(0)}{dx^5} = 5! = 120$, with digit sum 3

3. (b): The angle between any two planes is $\frac{\pi}{3}$. The planes form an equilateral prism. The planes $x + y = 1$ and $y - z = 2$ intersect along the line $\frac{x-1}{-1} = \frac{y}{1} = \frac{z+2}{1}$. The height of the prism, h is the distance of the point $(1, 0, -2)$ from the plane $x + z = 3$



$\therefore \quad h = \frac{|1-2-3|}{\sqrt{2}} = 2\sqrt{2}$

Area $= \frac{h^2}{\sqrt{3}} = \frac{8}{\sqrt{3}}$

4. (a): $x^3 + ix^2 + 2i = (x - i)(x^2 + 2ix - 2) = 0$

$\therefore \quad x = i, 1 - i, -1 - i$

$\Delta = \frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & -1 & 1 \end{vmatrix} = \frac{|-4|}{2} = 2$

5. (d): $I = \int_0^{\pi} x^2 \sin 6x\,dx = \left|\frac{x^2\cos 6x}{-6} - \frac{2x\sin 6x}{-36} + \frac{2\cos 6x}{216}\right|_0^{\pi} = -\frac{\pi^2}{6}$

6. (a, b, c, d):

$(1 - x)^{10} = C_0 - C_1x + C_2x^2 - C_3x^3 + \ldots + C_{10}x^{10}$

$x^9(1 - x)^{10} = C_0x^9 - C_1x^{10} + C_2x^{11} - C_3x^{12} + \ldots + C_{10}x^{19}$

Integrating between 0 and 1, we get

$\frac{C_0}{10} - \frac{C_1}{11} + \frac{C_2}{12} - \frac{C_3}{13} + \ldots + \frac{C_{10}}{20} = \int_0^1 x^9(1-x)^{10}dx = \frac{9!10!}{20!}$

$\therefore \quad n = \frac{11\cdot12\cdot13\cdot14\cdot15\cdot16\cdot17\cdot18\cdot19\cdot20}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9}$

$= 2^3 \cdot 5 \cdot 11 \cdot 13 \cdot 17 \cdot 19$

7. (c):

1, 6, 11, 16, 21
2, 7, 12, 17, 22
3, 8, 13, 18, 23
4, 9, 14, 19, 24
5, 10, 15, 20, 25

The number of ways when $x^2 - y^2$ is divisible by 5

$= 5\binom{5}{2} + 2\binom{5}{1}^2 = 100 \quad \therefore$ Probability $= \frac{100}{\binom{25}{2}} = \frac{1}{3}$

8. (d):

1, 8, 15, 22
2, 9, 16, 23
3, 10, 17, 24
4, 11, 18, 25
5, 12, 19
6, 13, 20
7, 14, 21

The number of ways when $x^2 - y^2$ is divisible by 7

$= 4\binom{4}{2} + 3\binom{3}{2} + 2\binom{4}{1}\binom{3}{1} + \binom{4}{1}^2 = 73$

$\therefore$ Probability $= \frac{73}{\binom{25}{2}} = \frac{73}{300}$

9. (9): The n functions are the onto functions from domain and codomain with 8 members each. They are bijections. $n = 8! = 40320$.

10. (c): a_1, a_2, a_3 are in G.P $\Rightarrow a_3 = a_2^2$

a_2, a_3, a_4 are in A.P. $\Rightarrow a_4 = a_2(2a_2 - 1)$

a_3, a_4, a_5 are in G.P. $\Rightarrow a_5 = (2a_2 - 1)^2$

a_4, a_5, a_6 are in A.P. $\Rightarrow a_6 = (2a_2 - 1)(3a_2 - 2)$

$a_5 + a_6 = (2a_2 - 1)(5a_2 - 3) = 198 \Rightarrow a_2 = 5$

$a_7 = (3a_2 - 2)^2 = 13^2 = 169$

P. $a_8 = (3a_2 - 2)(4a_2 - 3) = 13 \cdot 17 = 221$

Q. $a_9 = (4a_2 - 3)^2 = 17^2 = 289$

R. $a_{10} = (4a_2 - 3)(5a_2 - 4) = 17 \cdot 21 = 357$

S. $a_{11} = (5a_2 - 4)^2 = 21^2 = 441$

Solution Sender of Maths Musing

SET-146

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SET-147

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The best questions and their solutions will be printed in this column each month.

1. Show that $\tan\frac{\pi}{16}=\sqrt{4+2\sqrt{2}}-(\sqrt{2}+1)$.

–Anchal Srivastava (M.P.)

Ans. We know, $\tan\theta=\frac{1-\cos 2\theta}{\sin 2\theta}$, where $\theta=\frac{\pi}{16}$ ...(i)

Also, $\sin 2\theta=\sqrt{\frac{1-\cos 4\theta}{2}}=\sin 2\theta\Big|_{\theta=\frac{\pi}{16}}=\sqrt{\frac{1-\cos\frac{\pi}{4}}{2}}$

$=\sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}}=\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$ ...(ii)

and $\cos 2\theta=\sqrt{\frac{1+\cos 4\theta}{2}}=\sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}$; $[\because \theta=\frac{\pi}{16}]$

$=\sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}}=\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$ ...(iii)

Using (i), (ii) and (iii), we get

$$\tan\frac{\pi}{16}=\frac{1-\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}}{\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}}=\frac{\sqrt{2\sqrt{2}}-\sqrt{\sqrt{2}+1}}{\sqrt{\sqrt{2}-1}}$$

$$=\frac{(\sqrt{\sqrt{2}+1})\,(\sqrt{2\sqrt{2}}-\sqrt{\sqrt{2}+1})}{(\sqrt{\sqrt{2}+1})\cdot(\sqrt{\sqrt{2}-1})}$$

$$=\frac{\sqrt{2\sqrt{2}(\sqrt{2}+1)}-\sqrt{(\sqrt{2}+1)^2}}{\sqrt{(\sqrt{2}+1)(\sqrt{2}-1)}}$$

$$=\frac{\sqrt{4+2\sqrt{2}}-(\sqrt{2}+1)}{\sqrt{2-1}}=\sqrt{4+2\sqrt{2}}-(\sqrt{2}+1).$$

2. If $u=\sqrt{z_1z_2}$, prove that

$$|z_1|+|z_2|=\left|\frac{z_1+z_2}{2}+u\right|+\left|\frac{z_1+z_2}{2}-u\right|.$$

–Ventakesh (W.B.)

Ans. R.H.S. $=\left|\frac{z_1+z_2}{2}+\sqrt{z_1z_2}\right|+\left|\frac{z_1+z_2}{2}-\sqrt{z_1z_2}\right|$

$=\frac{1}{2}\left|z_1+z_2+2\sqrt{z_1z_2}\right|+\frac{1}{2}\left|z_1+z_2-2\sqrt{z_1z_2}\right|$

$=\frac{1}{2}\left|(\sqrt{z_1}+\sqrt{z_2})^2\right|+\frac{1}{2}\left|(\sqrt{z_1}-\sqrt{z_2})^2\right|$

$=\frac{1}{2}\left|\sqrt{z_1}+\sqrt{z_2}\right|^2+\frac{1}{2}\left|\sqrt{z_1}-\sqrt{z_2}\right|^2$

$\{\because\ |z^2|=|z\cdot z|=|z|\,|z|=|z|^2\}$.

Now, for any two complex numbers α, β, we get

$|\alpha+\beta|^2+|\alpha-\beta|^2$

$=(\alpha+\beta)(\overline{\alpha+\beta})+(\alpha-\beta)(\overline{\alpha-\beta})$ $\{\because\ |z|^2=z\bar{z}\}$

$=(\alpha+\beta)(\bar{\alpha}+\bar{\beta})+(\alpha-\beta)(\bar{\alpha}-\bar{\beta})$

$\{\because\ \overline{(z_1\pm z_2)}=\bar{z}_1\pm\bar{z}_2\}$

$=\alpha\bar{\alpha}+\alpha\bar{\beta}+\beta\bar{\alpha}+\beta\bar{\beta}+\alpha\bar{\alpha}-\alpha\bar{\beta}-\beta\bar{\alpha}+\beta\bar{\beta}$

$=2|\alpha|^2+2|\beta|^2$

$\therefore\ \frac{1}{2}|\alpha+\beta|^2+\frac{1}{2}|\alpha-\beta|^2=|\alpha|^2+|\beta|^2$.

So, R.H.S. $=|\sqrt{z_1}|^2+|\sqrt{z_2}|^2=|(\sqrt{z_1})^2|+|(\sqrt{z_2})^2|$

$=|z_1|+|z_2|=$ L.H.S.

3. A man parks his car among n cars standing in a row, his car not being parked at an end. On his return he finds that exactly m of the n cars are still there. What is the probability that both the cars parked on two sides of his car have left?

–Majeed (Lucknow)

Ans. Clearly, his car is at one of the crosses (×).

$|\times\times\times\cdots\times\times\times|$

The number of the ways in which the remaining $(m-1)$ cars can take their places (excluding the car of the man) $={}^{n-1}C_{m-1}$ $\{\because$ there are $(n-1)$ places for the $(m-1)$ cars$\}$

The number of ways in which the remaining $(m-1)$ cars can take places keeping the two places on two sides of his car $={}^{n-3}C_{m-1}$

$\therefore$ required probability

$=\frac{n(E)}{n(S)}=\frac{{}^{n-3}C_{m-1}}{{}^{n-1}C_{m-1}}$

$=\frac{(n-3)!}{(m-1)!(n-m-2)!}\times\frac{(m-1)!(n-m)!}{(n-1)!}$

$=\frac{(n-m)(n-m-1)}{(n-1)(n-2)}$.

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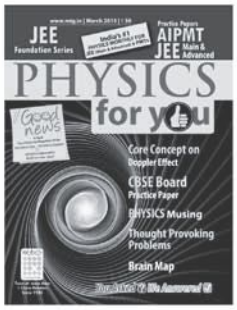

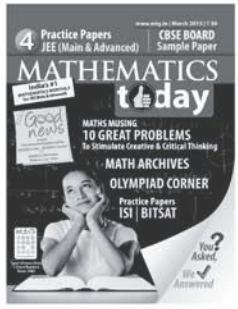

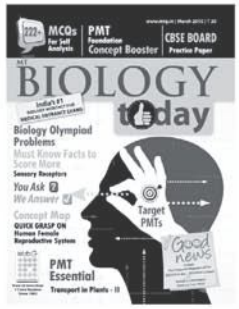

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